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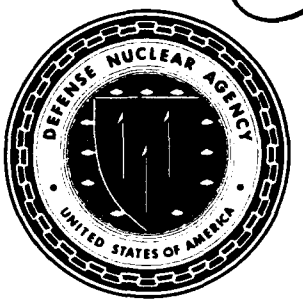

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The Meaning and Utility of Confidence

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This report presents an approach to establishing point estimates of system survivability along with associated confidence levels. It applies this approach to an example problem. The report also contains a probability tutorial.

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SUMMARY

This paper explores and discusses statistical methods and procedures that may be applied to validate the survivability of a complex system of systems that cannot be tested as an entity.

Confidence is a much-used and much-abused concept. We employ it here in conjunction with the capability to quantify system-level performance measures.

Procedures exist to develop system-level performance (e.g., survivability) distributions from corresponding performance distributions for piece-parts, components, and subsystems so that confidence statements about system performance can be made. Monte Carlo simulation is the method proposed to develop the system distribution from the component distributions using a system model that registers the logical interactions of the components to perform system functions. The system distributions permit developing point estimates and confidence intervals and bounds. This paper defines the terms and procedures for applying this technique.

Two principal issues surface in applying this approach to develop the survivability of large untestable systems of systems. The first is developing and validating the component survivability distributions. The second is that no general software is available to develop the system models and perform the simulations, although modifications to existing software to do so may be a feasible and attractive option.

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ESTIMATING SURVIVABILITY AT THE SYSTEM LEVEL

1.1 TOP-LEVEL STATEMENT OF THE PROBLEM.

Survivability is the probability that a component, subsystem, or system continues to function during and after being subjected to stressing threat environments, e.g., a nuclear detonation. System survivability estimates for complex systems are developed the same way that other system-level probabilistic performance measures (e.g., reliability, availability, maintainability) are developed - by defining events and estimating their probabilities of occurrence for piece parts, component boxes, and subsystems where test data are available to make such estimates, then combining this information in a system-level model to develop similar estimates for the system.

Validation includes tying survivability estimates to demonstrable test results showing the basis for survivability distributions and quantifying the associated uncertainties with probability intervals and confidence bounds. The concept of confidence bounds originated in classical statistics where the results of numerous identical tests are used to make statements about how certain we are that we know the value of specified performance parameters. Classical statistical confidence bounds can be placed on the performance of any system or system element that can be tested as an entity, and the tighter the bounds, the greater number of tests required.

By system we mean any collection of piece parts, components, and subsystems connected in some way to perform prescribed functions. The ultimate system of interest herein is a large complex system comprising a number of major systems, i.e., a system of systems. We use the word system to describe collections of sub-elements at any level of complexity.

To develop system-level performance estimates, we develop an event model of the system to the level of detail for which we have legitimate data. If the system as a whole can be tested repeatedly in the specified environment, then no integrating model is required. The system is treated as an entity. The data are legitimate and applicable, and either a performance measure probability distribution or confidence bounds upon system performance can be developed.

Usually, complex systems cannot be tested as entities because of excessive cost, or because the threat environments cannot be generated or the system operated without inordinate hazard to people and property. Consequently, the attractive assessment option of testing the system as a whole is rarely viable.

Instead, almost universally, we are forced to structure tests, to extract data, and to develop survivability distributions and estimates with associated uncertainty measures on piece parts, component boxes, and subsystems, where risks and costs are affordable. Then, by developing appropriate system models and making necessary assumptions, we combine this information at the element level to develop system-level survivability distributions and estimates with associated measures of uncertainty (confidence).

It is important to understand that the statistical models we discuss here do not provide the values of physical parameters involved in system operation such as

voltages, currents, accelerations, forces, pressures, temperatures, and displacements. Instead, such models treat element events whose probabilities of occurrence have been derived from knowing or estimating the actual physical parameters and their stressing effects upon components. Such component survivability events are defined as "Event A is the event that component A survives a specified threat." The probability of occurrence of event A, the survivability of component A, is determined from test data or from computer simulation of the stress imposed, and the resultant physical damage, if any, upon component A. The likelihood of surviving is often determined by judgment. Component fragility functions relating the probability of killing (Pk) a component, or rendering it inoperable, to the stress level imposed may be developed from test data, in which case, given the stress level, the Pk is known and the survivability estimate is 1 - Pk. The stress levels themselves from specific environments will also have uncertainty distributions, so even at the most elemental level survivability will be expressed as a distribution, where survivability is a random variable, not a fixed, but unknown, parameter.

Given that such survivability distributions have been, or can be, developed at the piece part or higher level, probabilistic system models (e.g., GO models, event trees, or fault trees) can be developed to estimate system survivability. These models are also utilized to quantify the effects of systematic uncertainties and random errors at the piece part, component, and subsystem levels on system-level survivability. This can be done deterministically (point estimates) or by Monte Carlo simulation. Simulation is a very attractive approach because it is applicable at any level of complexity and fewer simplifying assumptions are required to employ it. With the order of magnitude increases in computer speeds and capabilities and resultant cost decreases within the last decade, today simulation, where samples from the distributions for components and subsystems are integrated in a system-level model to develop system-level distributions, is considered to be a tractable, desirable, and perhaps the only feasible method for addressing this problem.

Developing validated survivability distributions for system elements for postulated threat environments is the principal effort required to employ simulation to develop system-level survivability estimates with confidence.

A major complexity in estimating survivabilities of components is introduced when there are statistical dependencies in the amounts of damage inflicted by imposing several simultaneous, or near-simultaneous, environments and where synergistic damage results. In this case, some type of joint fragility function is required to be able to predict the probability of component kill. Such relationships are developed by testing, by computer simulation, or by artificial intelligence models. The capability to develop or approximate these survivability distributions is a fundamental requirement in being able to conduct quantitative system-level survivability assessments.

Another difficult complexity is cumulative damage from several sequential exposures of varying magnitudes from different sources. Here, time-phased survivability estimates may be required along with the capability of simulating performance for cumulative damage levels.

In developing the component survivability distributions there are two major, and different, sources of uncertainty. These are systematic uncertainties and random errors.

Random error is irreducible experimental scatter. Electromagnetic radiation of the same wave lengths and intensities impose varying stresses on components as measured when conducting repeated tests on supposedly identical equipment because of minute physical differences. This irreducible variation gives rise to random error stress, damage, and survivability distributions that quantify the nature and extent of such random variations.

Systematic uncertainties are parameters that have single true values which are unknown, but which can be known by additional testing and analysis. For example, the exact nature of the stresses imposed on electrical equipment from a specified nuclear weapon detonated at various locations could conceivably be determined by conducting a number of tests. In real life, however, it is manifestly impossible to conduct more than a few such tests. Using their results we then extrapolate the nature of the environments expected at various other locations, but with unknown biases - systematic uncertainties.

We assume that the elemental survivability distributions that will be used to develop system survivability distributions will include the effects of both systematic uncertainties and random errors. Such distributions are developed by sampling from the known or postulated distributions of both types of uncertainties with equal weighting and frequency. The resultant distributions correctly reflect the extent of uncertainties from all sources of error. Often, the object in performing system-level survivability assessments is to determine how to reduce uncertainties and increase survivability - shift the distributions to the right. With a developed system model this can be done by performing sensitivity analyses - varying the parameters of various elements and determining which affect system survivability most.

1.2 A SUGGESTED APPROACH.

We state at the outset, that even for relatively small systems, there is no mathematically rigorous analytical method for determining the probability distributions of system performance from the known probability distributions of the constituent components, except for some very restricted cases, e.g., all components are in series and have identical performance distributions. Consequently, in light of increased computer capabilities, we see Monte Carlo simulation as being the only viable approach for conducting assessments for complex systems. However, to our knowledge, at present there is no generic software to perform this task.

Over the years, several approximating schemes to place confidence bounds on system performance based upon component information have been devised, e.g., Lindstrom-Madden, Maximus, Myhre, Easterling, most of which require the postulated decomposition of the system to a linear array. Because the problem is so difficult, usually requiring hand manipulation and judgment, and because there is virtually never sufficient test or use information to fully develop the underlying distributions, confidence bounds are seldom stated for complex system performance, or, if so, have been derived from a limited number of tests of the system as a whole, or by approximation and with numerous assumptions and judgments about component performance and the validity of data. While using simulation will not obviate all of these difficulties, it will make the estimation process legitimate, understandable, and tractable (although this remains to be demonstrated).

The survivability performance measure is, in many ways, more difficult to assess than others because the components, subsystems, or system must be subjected to realistic stressing environments (which are often severe, hazardous, or catastrophic)

to develop quantitative survivability distributions and estimates. For postulated nuclear weapon environments, for example, no real-life testing is possible, and all data must be developed from limited numbers of underground tests (UGTs) and aboveground tests (AGTs) that approximate or simulate such environments, and this too on only piece parts, or at best, upon small portions of systems. Then this data must be extrapolated by judgment and computer simulation to probable effects upon larger systems with increased uncertainties.

1.3 AN ABBREVIATED TOP-LEVEL EXAMPLE.

Since simulation appears to be the only viable way to conduct system-level performance assessments, we present an abbreviated top-level example showing how Monte Carlo simulation can be used to develop system-level survivability distributions and confidence functions.

The diagram in Figure 1-1 portrays the present concept of an early National Missile Defense (NMD) system. We will use it as a baseline prototype system to demonstrate the survivability modeling and estimation procedure we suggest for quantifying complex system survivability and placing confidence bounds upon it.

An actual architecture will include larger numbers of sensors, radars, interceptor farms, and interceptors, and more detailed descriptions of equipment and communication devices. For the purpose of this example this abbreviated system will suffice.

The operational concept of the early NMD of Figure 1-1 is that the orbiting DSP or Brilliant Eyes sensors immediately detect a threat launch. For our purposes we postulate that of 20 or so such orbiting sensors only six can "see" the launch, and to fully perform their function of launch detection, track, threat discrimination, commit of ground based interceptors, and subsequent kill assessment, at least four of the six relevant sensors must remain functional throughout the attack, despite a prior or coincident nuclear detonation in the battle space.

Upon detection of another threat launch, CINCSPACE in the command center at Cheyenne Mountain, Colorado Springs, CO, probably already has National Command Authority (NCA) authorization to intercept, but continues to keep the NCA informed of the developing threat. The BMEWS/Pave Paws early warning radar systems confirm the threat, but they may be degraded or may not have survived the prior detonation.

CINCSPACE commands the GPALS Operations Center to engage the threat. The GPALS Operations Center plans and tasks the Ground-Based Surveillance and Tracking System (GSTS), then launches a GSTS missile with an IR probe and processes its incoming data. As the threat approaches the U.S., GSTS hands-off data to the Ground Based Radar (GBR) which acquires and tracks the follow-on threat. Of course, the GSTS, the GPALS Operations Center, and the GBR may not have survived the prior detonation.

The GPALS Operations Center plans the engagement and tasks specific exoatmospheric interceptors, then launches them from a GBI launch site. On the basis of continuing GBR data, the Operations Center transmits in-flight target updates to the interceptors. We postulate the deployment of one, two, or at most three interceptors. Their performance (i.e., survivability), which also depends upon the survivability of other system elements, is the final measure of system survivability to the postulated nuclear detonation in the battle space.

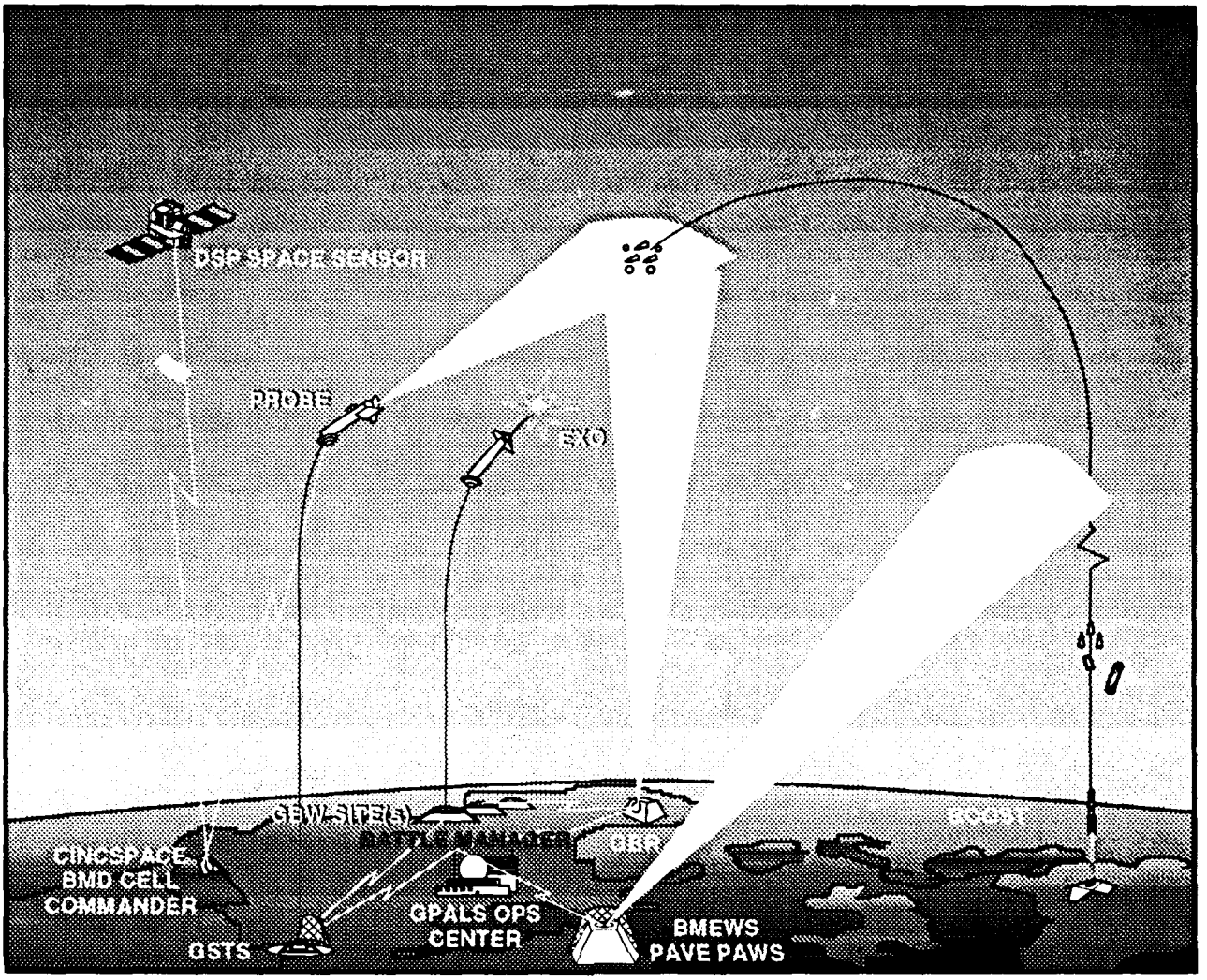

Figure 1-1. Early NMD operational concept.

1.3.1 A Simple Model.

There are several methods that could be used to develop a system level model of this early NMD system. Perhaps the most general approach that could be used would be to write logic event or algebraic probabilistic equations expressing the functional relationships and probabilities of system performance. Developing success event trees or fault trees for undesirable system events is another general method that could be applied. For the purpose of this paper, we develop a GO survivability logic model of the early NMD system (Figure 1-2) showing how the various subsystems interact to develop the overall system survivability. For each of the sub-elements of the system we assume the existence of a survivability distribution that reflects both the systematic uncertainties and the random errors inherent in estimating the survivability of that sub-element. These survivability distributions may well have been derived from the simulation and combination of similar survivability distributions for smaller system elements.

The GO model of Figure 1-2 represents each of the elements or subsystems of the early NMD system with a pair of "type"-"kind" numbers separated by a hyphen. The "type" number captures the logical essence of the component and refers to one of 17 defined logical operators in the GO methodology. The associated "kind" number is simply the sequential number in an array that references the probabilities with

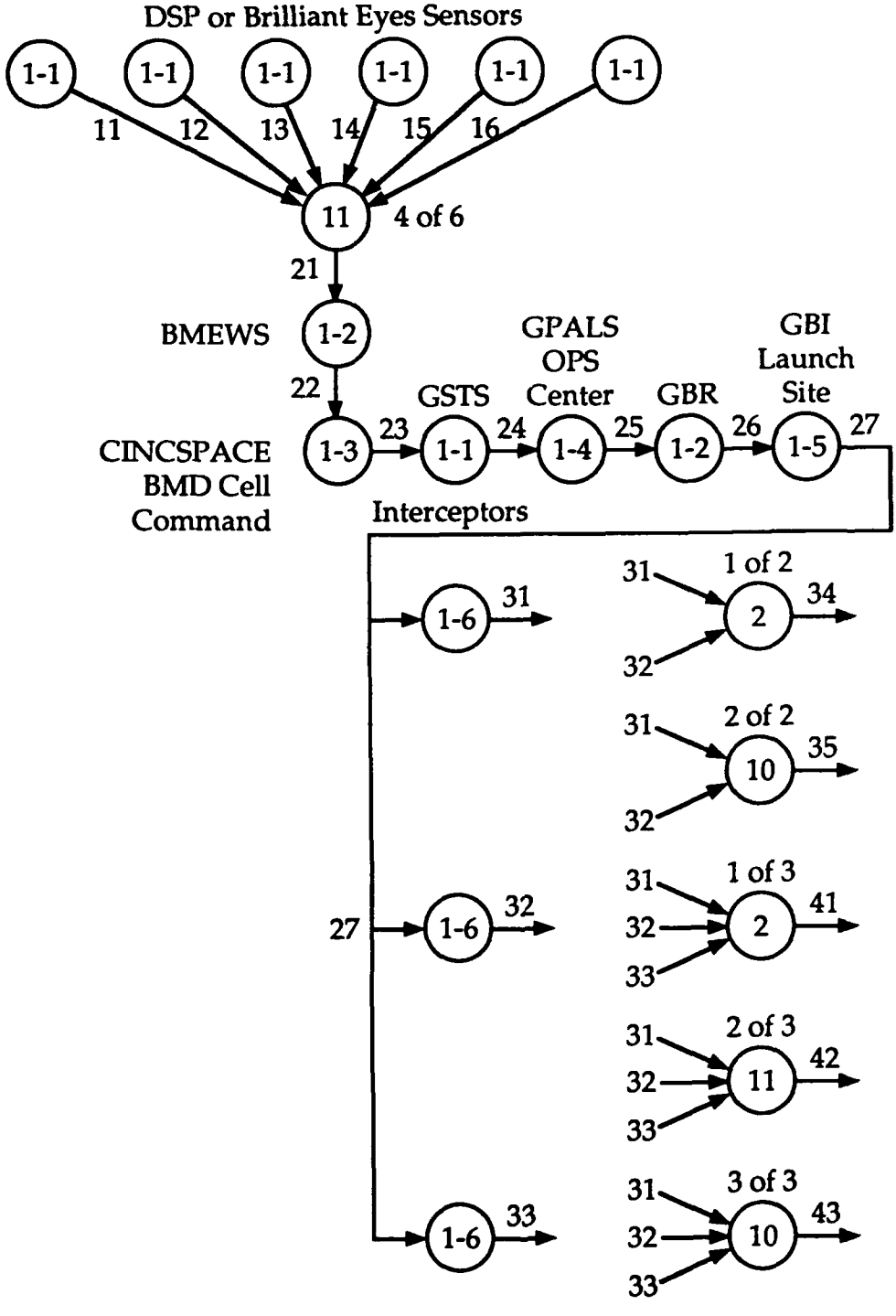

Figure 1-2. GO logic model of early NMD operational concept.

which that component takes its several operational states - e.g., good, bad, premature. The recorded input and output "signals" (a carryover from electrical schematics) are really discrete random variables that take pre-defined values representing success or failure. For this example we permit the random variables to take only two values - 0 for success, 1 for failure. Events are then expressed as a signal taking a value. For example, the success event that DSP satellite number 1 operates properly when we exercise the model is expressed as the event that signal 11 takes value 0, or simply 11_0.

Elements of the model that simply represent logical operations - "and" gates, "or" gates, or "m out of n" - gates have no associated probabilities, and, consequently, no associated kind numbers. For example, the type 2 operators represent "or" gates; the type 10 operators represent "and" gates; and the type 11 operator represents an "m out of n" gate. An "m out of n" gate expresses the number of inputs that must be good to produce a good response. In our model the type 11 operator expresses the fact that at least 4 of the 6 satellite sensors must be operating properly to provide the necessary information for success.

The GO system survivability logic model is developed to represent the manner in which the various sub-elements must function and interact for the system to survive and perform its intended mission given a postulated nuclear detonation within the general vicinity of a theater defense system including the associated DSP or Brilliant Eyes sensors, the GSTS, the GPALS Operations Center, the Ground Based Radar, and the Ground Based Interceptors and Launch Site. Presumably, the Command Center is not affected by this scenario, being both remote and extremely hard. Our purpose is to develop the survivability of this system from a nuclear detonation in a specified location. This will be accomplished by developing (in this case assuming) the survivability distributions for each major subsystem, then sampling from these distributions and combining the estimates in the system GO logic model, to develop overall system survivability distributions by simulation. This can be done at any level of sophistication, from piece parts to major subsystems.

In a real-life application the subsystem survivability distributions would be developed from knowledge or simulation of the responses of piece parts and components to the specified environment. In this example we assume the existence of the survivability distributions. In each case we have assumed that they are beta probability distributions, but there is nothing magic about beta distributions nor are there any restrictions on the type of distributions that can be employed, except that the range of the random variable, survivability, which is a probability, must lie between 0 and 1 (a nice feature of beta distributions). Beta distributions also have the capability to adopt a multitude of forms by varying the two parameters which can be directly related to the numbers of successes and failures in repeated trials. Consequently, beta distributions can represent any number of real or perceived distributions and thus find common utility in real-life models.

A beta density function with parameters "a" and "b" is defined by the expression:

$$\begin{aligned} f(x) &= kx^{a-1}(1-x)^{b-1} \qquad 0 < x < 1 \\ &= 0 \qquad\qquad\qquad\quad \text{elsewhere,} \end{aligned}$$

where $k = (\Gamma(a+b)/\Gamma(a)\Gamma(b))$ and Γ is the well-known Gamma function. The mean and standard deviation are:

$$\mu = a/(a + b);$$
$$\sigma^2 = ab/((a + b)^2(a + b + 1)).$$

If parameters a and b are both 1, then the beta distribution reverts to the simple uniform distribution over the unit interval. The parameters a and b can also be related to the numbers of success and failures in n trials.

Table 1-1 records the GO type-kind numbers (Kaman Sciences Corproation, 1983, 1985) that identify the sub-elements, the beta parameters that identify the postulated survivability distribution ascribed, the resultant means and variances of each distribution, and the names of the associated sub-elements. Five such distributions are postulated. No distribution is defined for the Command Center because we postulate no effect upon its survivability due to the remote nuclear detonation.

Table 1-1. Early NMD survivability distributions.

Type-Kind Numbers	Beta Param	Mean	Var.	Sub-Elements
1-1	8,2	0.8000	0.0145	Brilliant Eyes, DSP, & GSTS
1-2	88,2	0.9778	0.0002	BMEWS & GBR
1-3	None	--	--	Command Center-CINCSPACE
1-4	98,2	0.9800	0.0002	GPALS Operations Center
1-5	78,2	0.9750	0.0003	GBI Launch Site
1-6	18,2	0.9000	0.0043	Interceptors

The beta density functions and survivability (cumulative probability) distributions for each of the five sets of parameters recorded above are depicted in Figures 1-3 to 1-12.

As noted above the survivability distributions of each of the six orbiting sensors are postulated to be beta distributions with parameters 8,2. This might be representative of the results of 8 tests, 7 successes and 1 failure, so parameter a = 7 + 1 and parameter b = 1 + 1. Since at least four of these six sensors must remain functional to perform the mission of detecting additional missile launches, discriminating threats, providing data to guide interceptors, and assessing kills, this logic is accommodated with a GO logic 4 out of 6 type 11 operator. While we have assumed that the survivability distributions of all six sensors are identical, the survivability of each is statistically independent of the others in the simulation.

Each of the succeeding early NMD system elements necessary for system operation - the BMEWS system, the Command Center, GSTS, GPALS Operations Center, GBR, and GBI Launch Site are modeled as shown in Figure 1-2 and their operation and performance is logically linear. The single launch site launches one, two, or three interceptors (the type 1-6 GO elements) according to the firing doctrine producing "signals" 31, 32, and 33 (events) representing the performance (survivability) of each of the three interceptors. These are final outputs from the

model. Their distributions will be identical since we have postulated identical survivability distributions for them and will be developed as a function of all the necessary sub-elements in the system. Hence, the interceptor survivability distributions are not statistically independent because each depends upon all other system elements to the same degree. If one or more of the interceptors were in-flight and close to the detonation when it occurred, survivability would be much less feasible than postulated. In that case different distributions would be required.

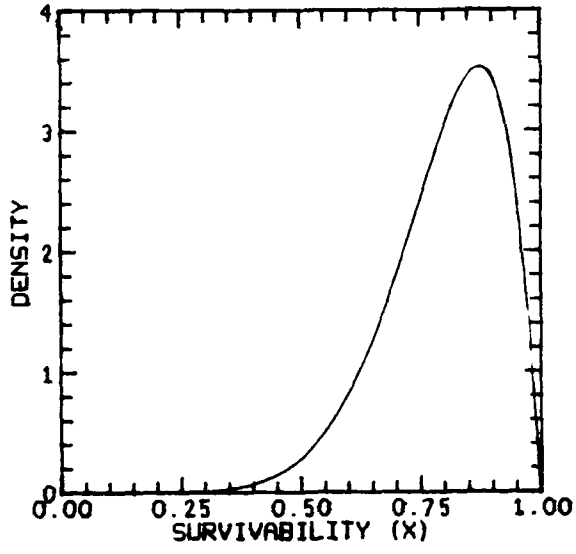

Figure 1-3. Density function beta(8,2).

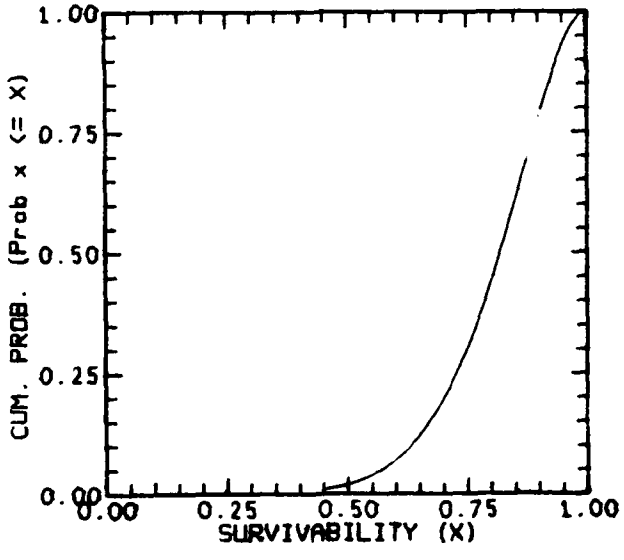

Figure 1-4. Survivability distribution beta(8,2).

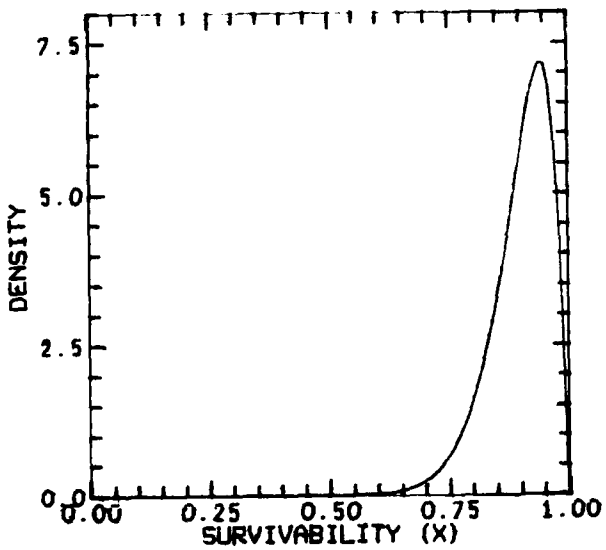

Figure 1-5. Density function beta(18,2).

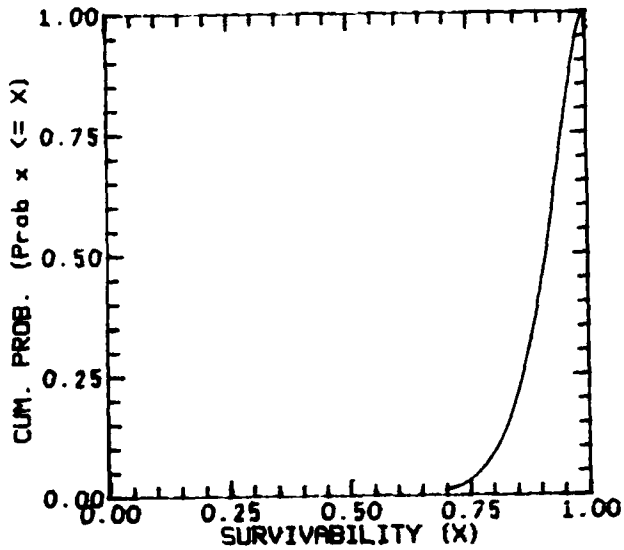

Figure 1-6. Survivability distribution beta(18,2).

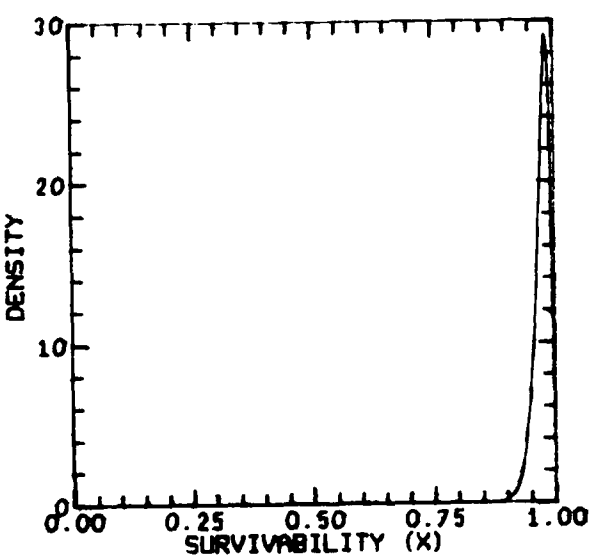

Figure 1-7. Density function beta(78,2).

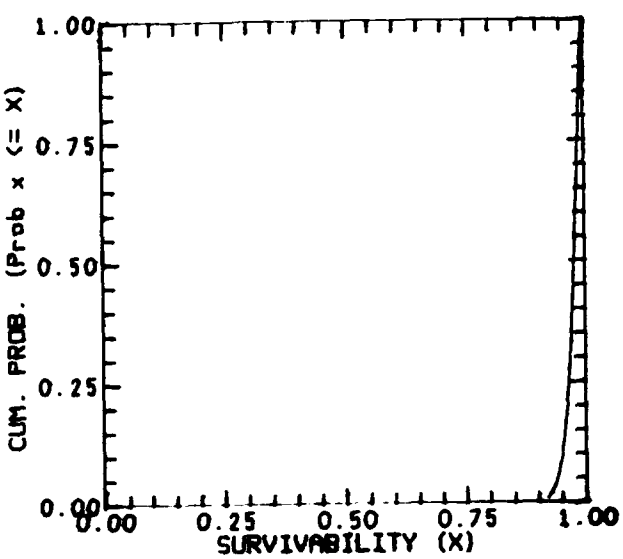

Figure 1-8. Survivability distribution beta(78,2).

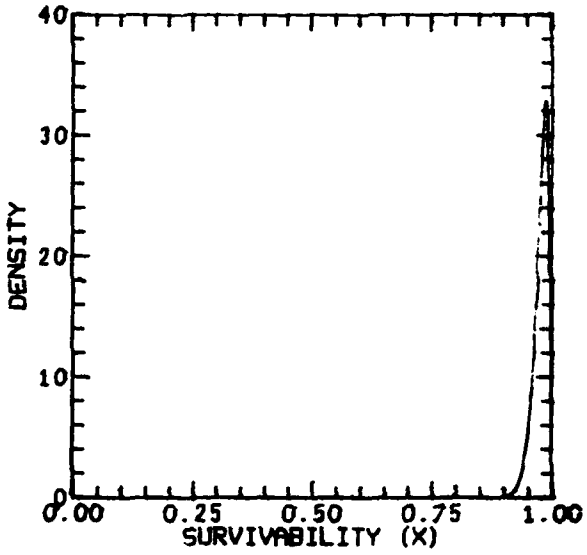

Figure 1-9. Density function beta(88,2).

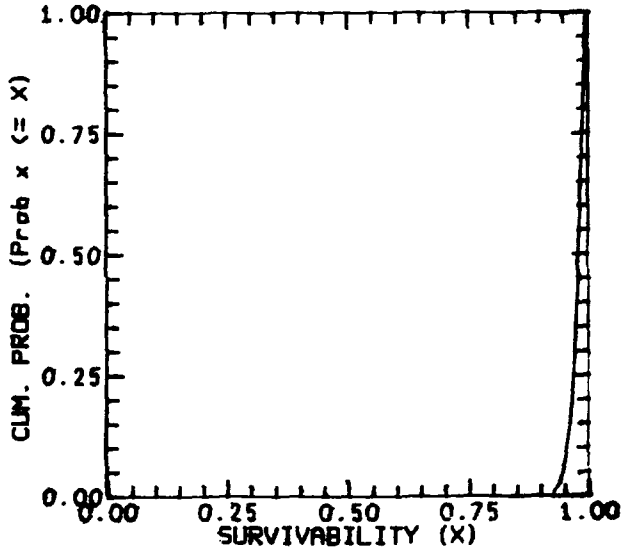

Figure 1-10. Survivability distribution beta(88,2).

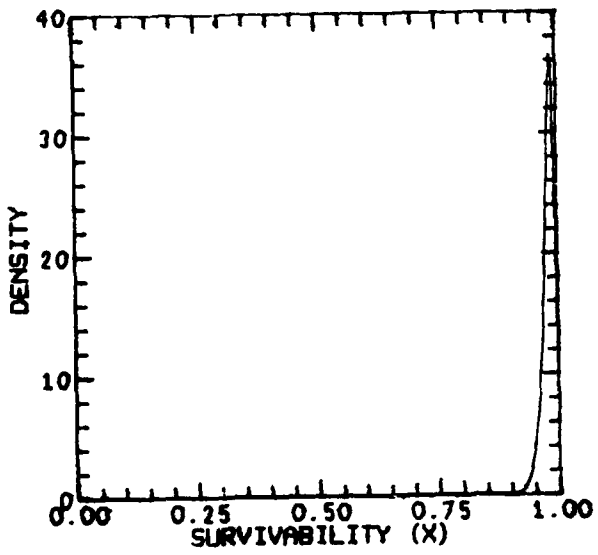

Figure 1-11. Density function beta(98,2).

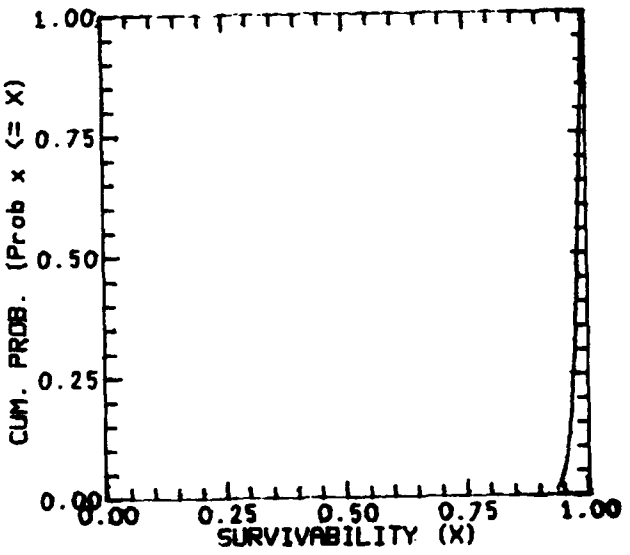

Figure 1-12. Survivability distribution beta(98,2).

In addition to the single interceptor outputs, we add additional logic to determine the probability distributions for several events, i.e., for the survival of a single interceptor assuming that only one is launched (signal 31); for 1 of 2 (signal 34) and for both interceptors (signal 35) assuming that two are launched; and for 1 of 3 (signal 41), 2 of 3 (signal 42), and all three interceptors (signal 43) assuming that three are launched.

Because we do not have a general tool to perform these types of simulations, the procedure employed to perform this simulation was to sample from each of the postulated survivability distributions for system elements, then either take the survivability estimates on each trial and hand-enter them into a GO data file and process them using the GO software in accordance with the system model of Figure 1-2 to derive system survivability estimates, or, alternatively, to write the system logic equations by hand and code them to perform the simulation. Both procedures are time-consuming, labor intensive, and probably impractical for assessing large complex systems.

We used both procedures to insure that the model was correct, then ran 1000 simulations to develop the six system survivability distributions defined above, one each for signals 31, 34, 35, 41, 42, and 43.

1.3.2 The Solution - Survivability Distributions And Confidence Functions.

The resultant survivability distributions are graphed in Figures 1-13, 1-14, and 1-15, Example Early NMD Survivability Distributions given that one, two, or three interceptors were launched. The sample survivability distributions portray the variability in the random variable "system survivability" for different events and firing doctrines (numbers of interceptors launched).

The distribution for all three interceptors surviving, given that three were launched is shown by the "all three interceptors" curve of Figure 1-15. This distribution has a mean of 0.48 and standard deviation of 0.10. The distribution for at least one of the three interceptors surviving, the "one of three interceptors" curve of Figure 1-15 has mean of 0.66 and standard deviation of 0.11.

Having developed these early NMD survivability distributions by simulation, we can use them to place confidence bounds on the probability of surviving a nuclear detonation. This can be done directly from Figures 1-13 to 1-15, but we prefer to invert the distributions and read confidence (1. - the cumulative probability) directly. Hence, we plot the several confidence functions for these survivability distributions in Figures 1-16, 1-17, and 1-18, example Early NMD Confidence Functions, given that one, two, or three interceptors were launched.

Table 1-2, Example Early NMD Survivability Estimates And Bounds, records the resultant parameters and bounds of all six distributions.

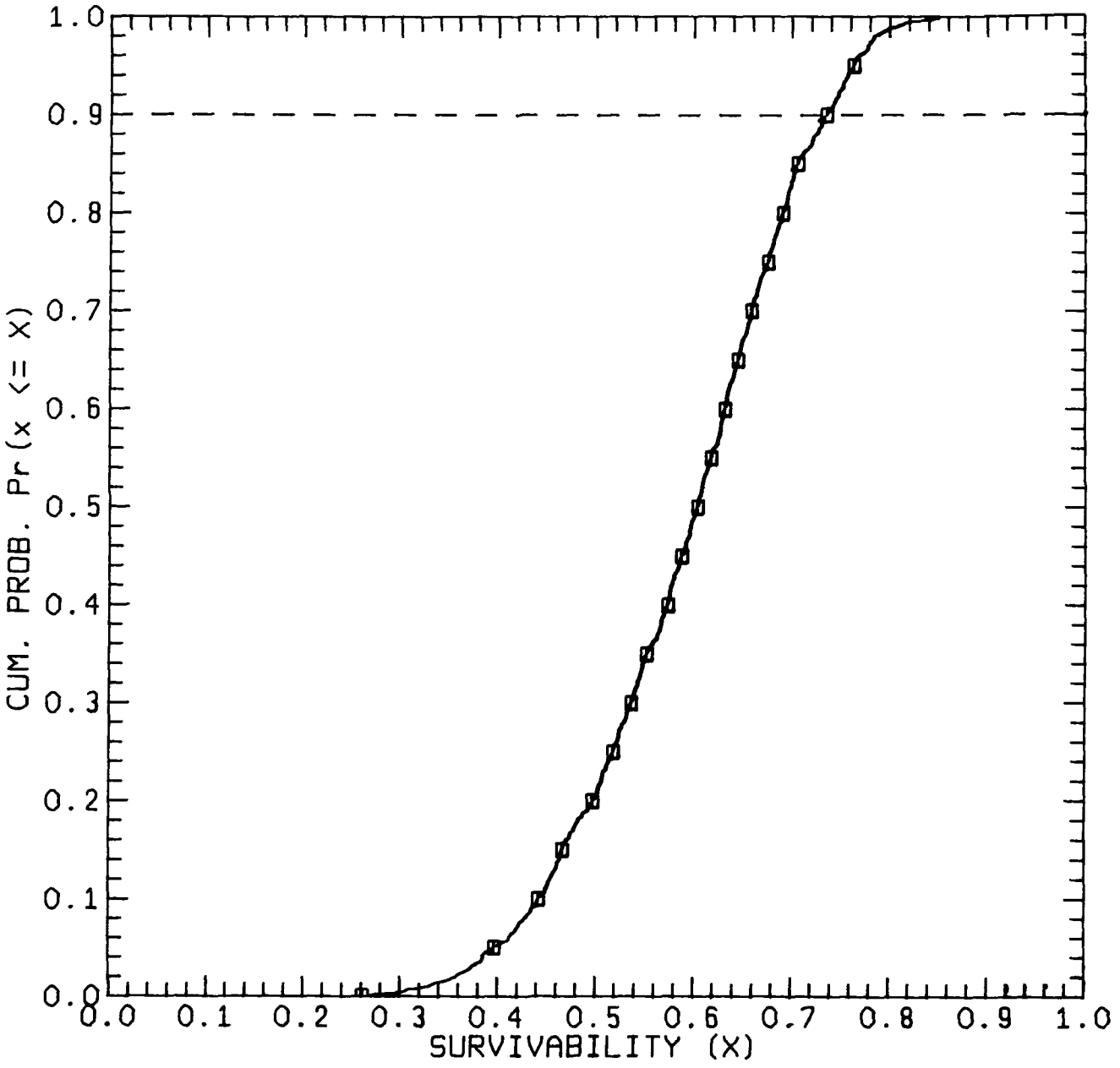

Figure 1-13. Example early NMD survivability distributions - single interceptor launched.

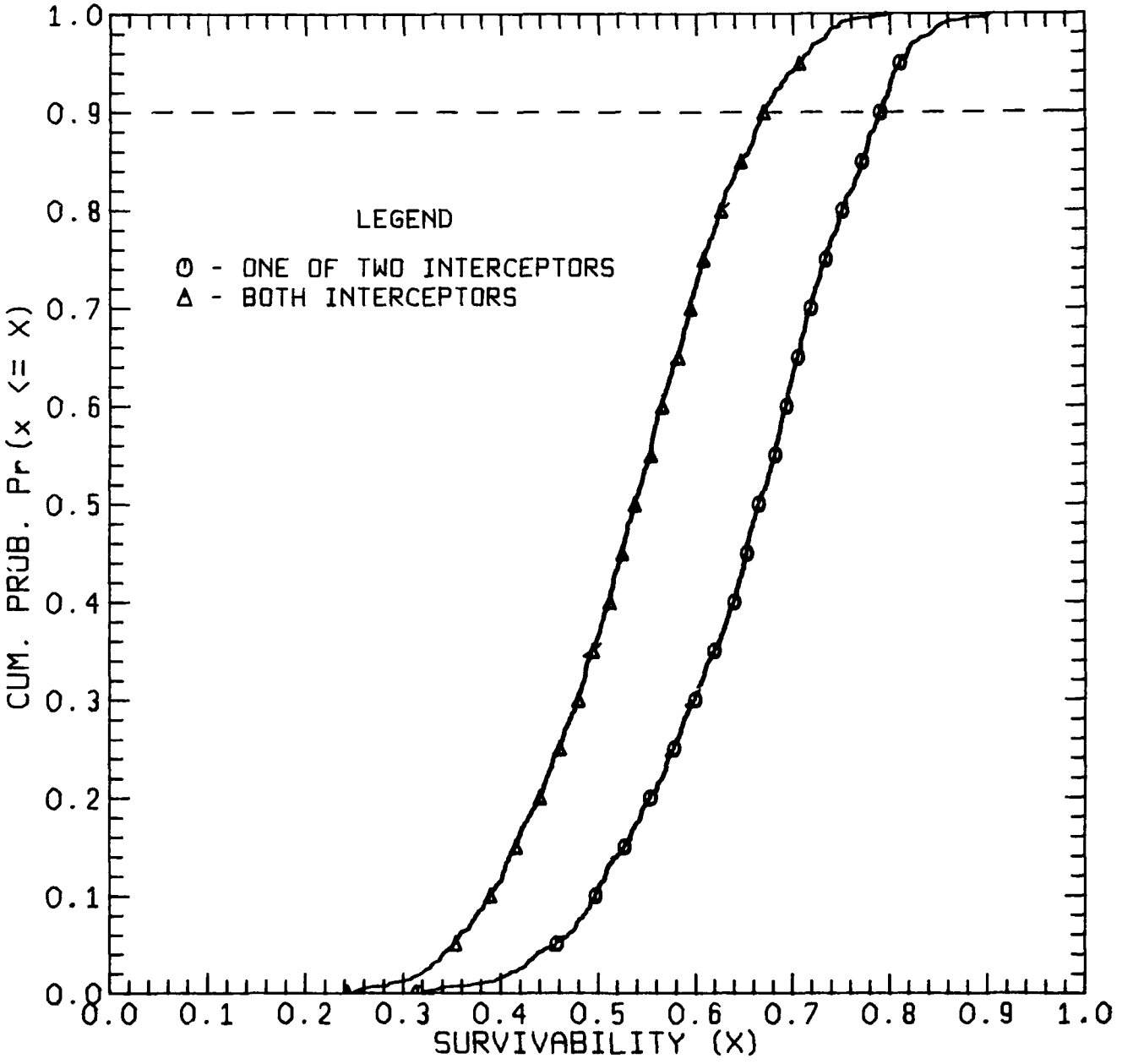

Figure 1-14. Example early NMD survivability distributions - two interceptors launched.

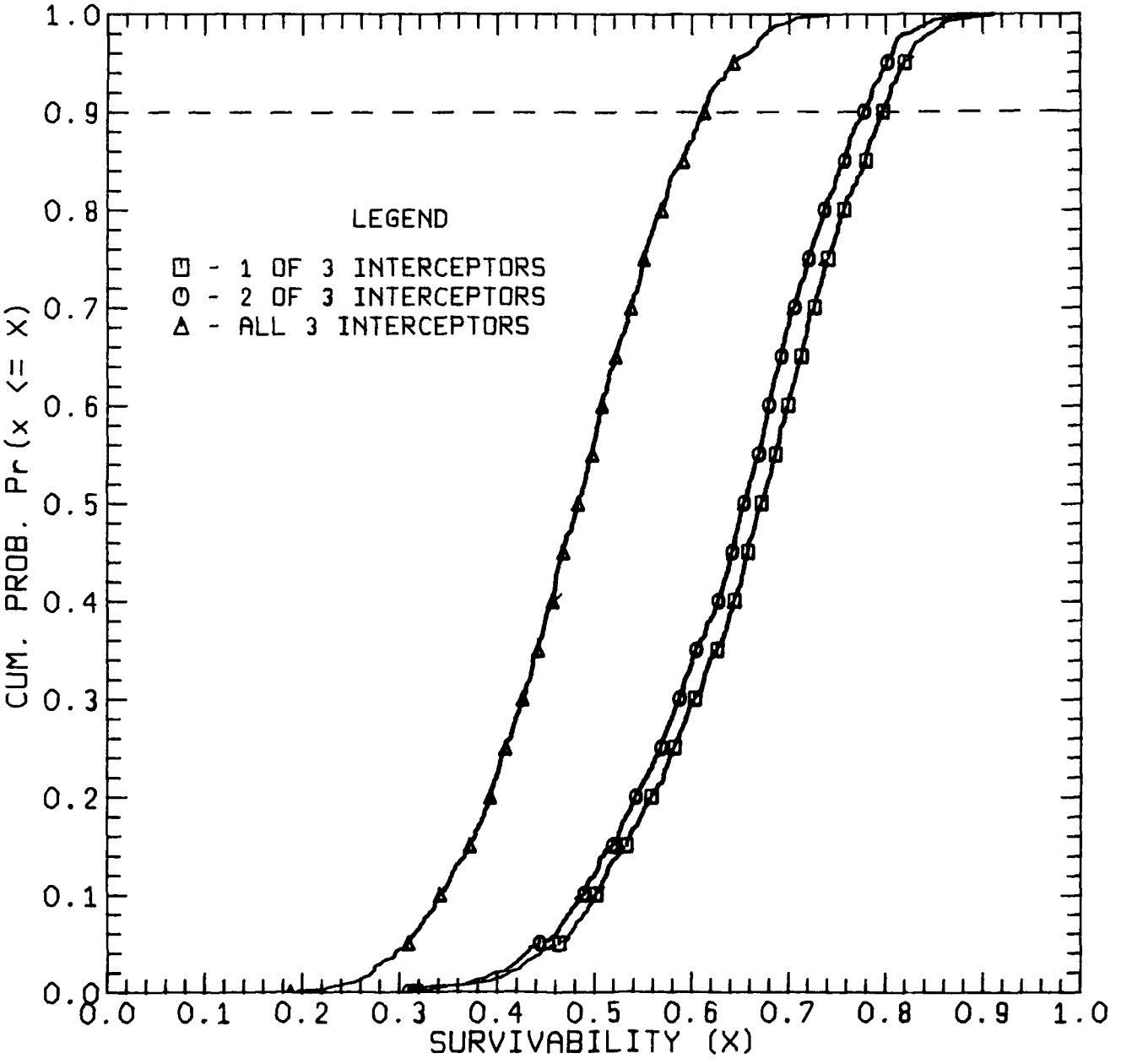

Figure 1-15. Example early NMD survivability distributions - three interceptors launched.

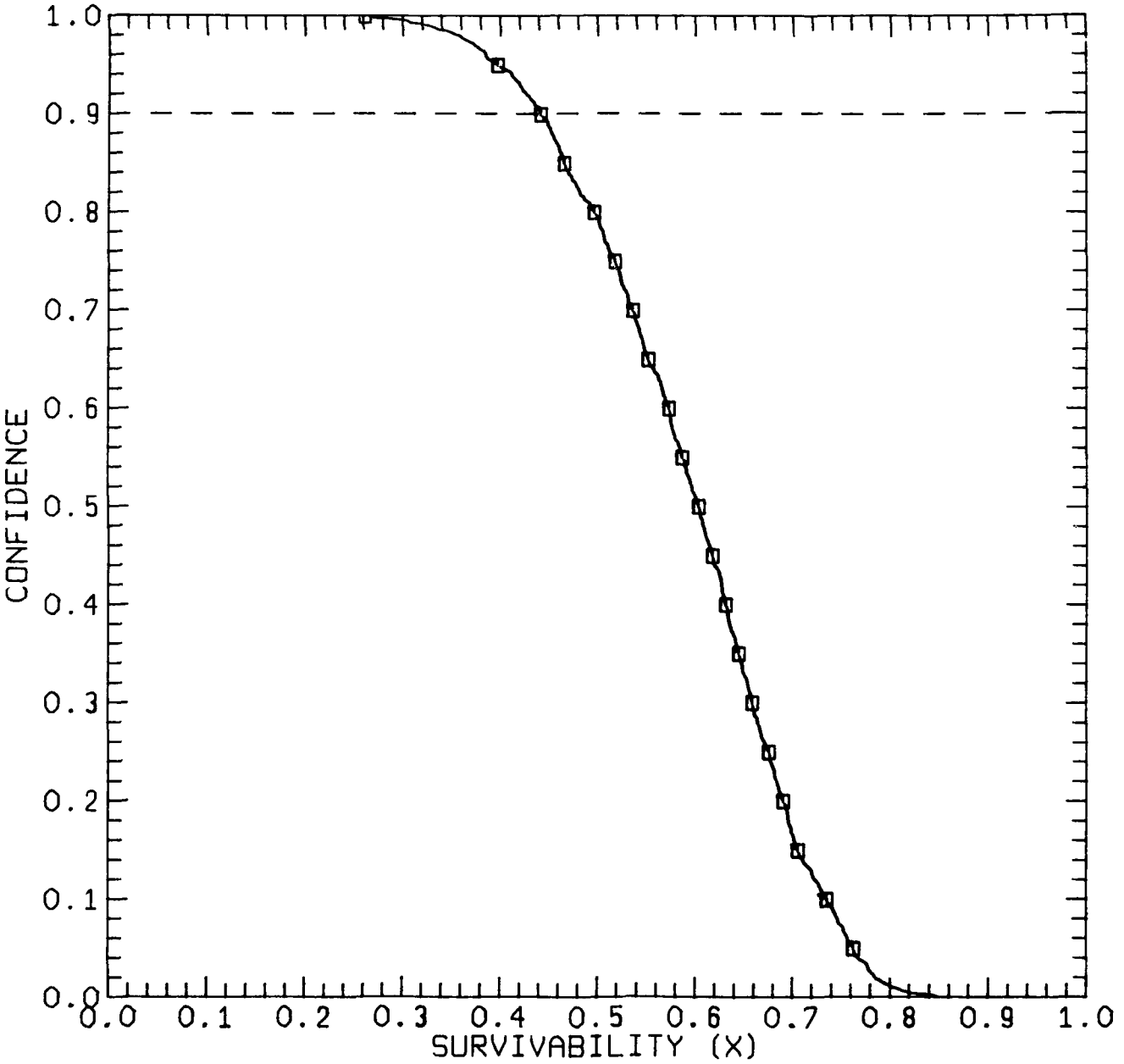

Figure 1-16. Example early NMD confidence function - single interceptor launched.

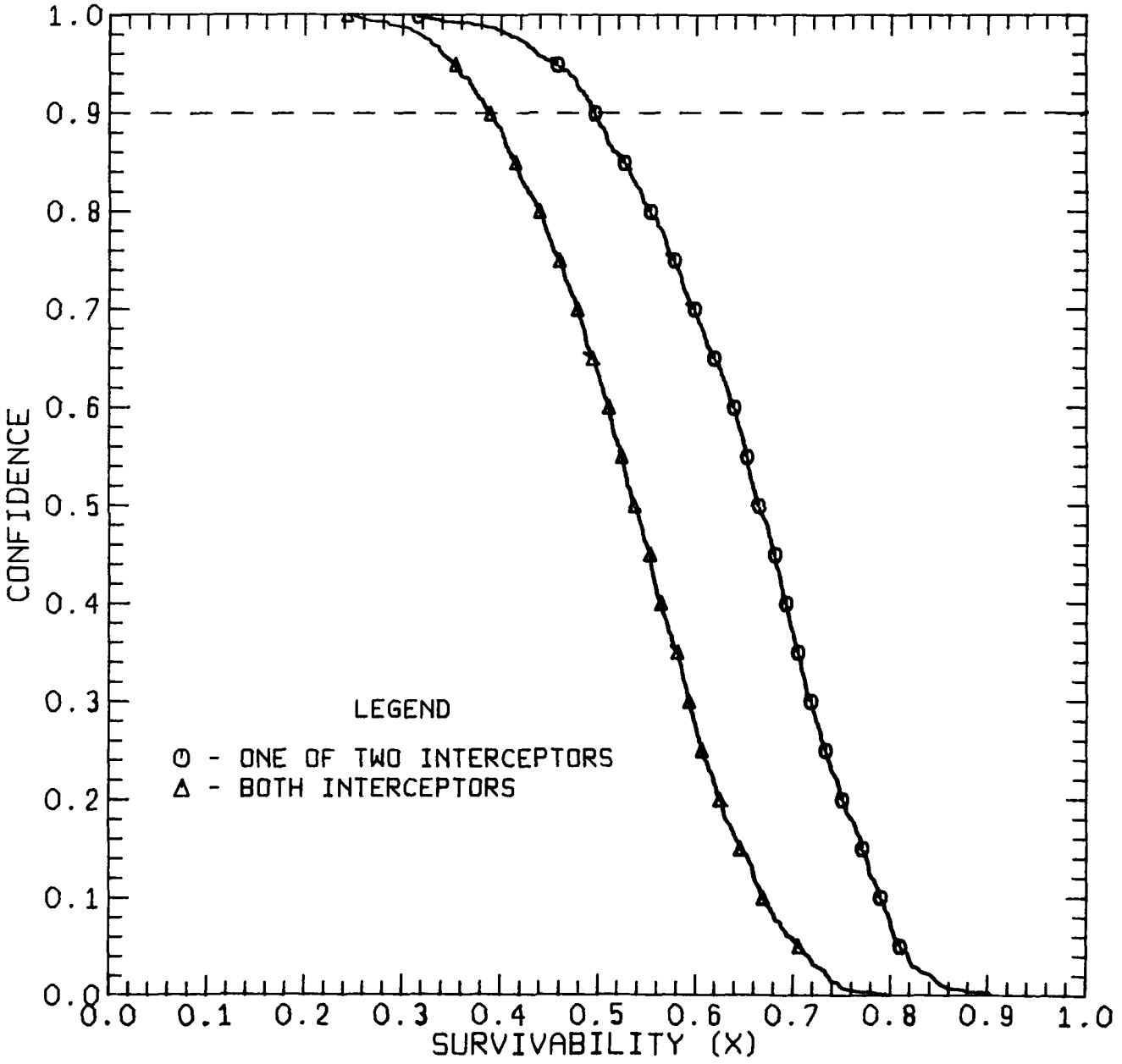

Figure 1-17. Example early NMD confidence functions - two interceptors launched.

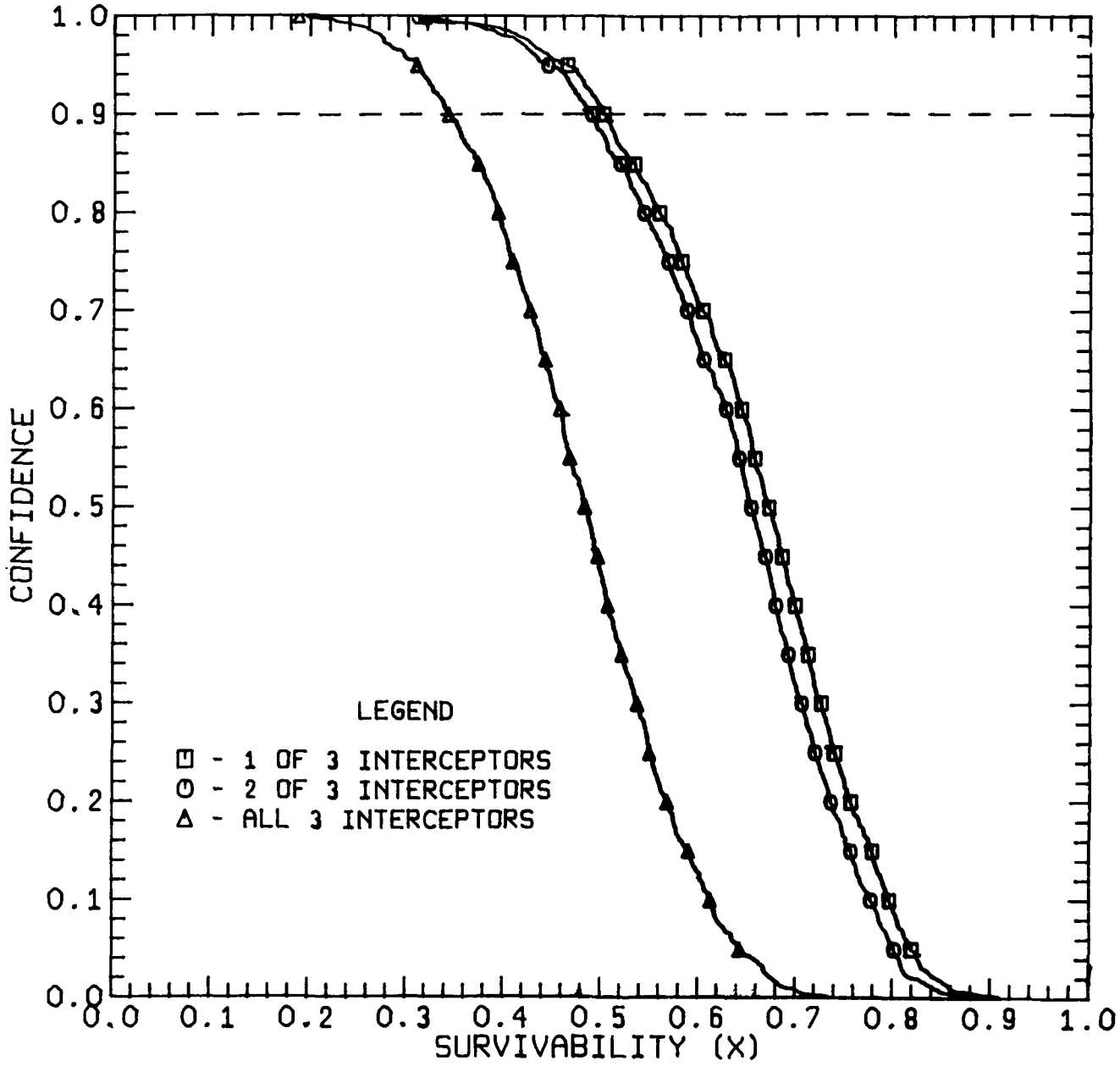

Figure 1-18. Example early NMD confidence functions - three interceptors launched.

Table 1-2. Example early NMD survivability estimates and bounds.

Case	Mean	S.D.	90% LCB	50% LCB
1 of 1	0.593	0.11	0.44	0.60
1 of 2	0.652	0.11	0.50	0.67
2 of 2	0.533	0.11	0.39	0.54
1 of 3	0.663	0.11	0.50	0.67
2 of 3	0.640	0.11	0.49	0.66
3 of 3	0.499	0.10	0.36	0.48

From the functions in Figures 1-16 to 1-18 we state that we are 90% confident that the survivability of a single interceptor given that only one was launched is greater than or equal to 0.44; that we are 90% confident that the survivability of at least one interceptor given that two were launched $\geq$ 0.50; and, that we are 90% confident that the survivability of all three interceptors given that three were launched $\geq$ 0.36, etc. Of course, having the entire distributions we can make confidence statements at other levels of confidence. For example, we can also state that we are 50% confident that the survivability of a single interceptor (given that only one was launched) $\geq$ 0.60, of at least one interceptor surviving given that two were launched $\geq$ 0.67, and that all three survive given that three were launched $\geq$ 0.48. Obviously, the higher the confidence the lower the bound.

A principal objective in performing system-level survivability assessments will be to improve survivability (i.e., move the distributions to the right) and to reduce uncertainty (that is, make the curves more nearly perpendicular). To portray the reduction of uncertainty we repeated the simulation above having eliminated all uncertainties in the GSTS, giving it a survivability point estimate of 0.80 (the mean of the former distribution). We portray the results in Figures 1-19 and 1-20. Figure 1-19 shows the new survivability distribution for a single interceptor and Figure 1-20 depicts the resultant confidence functions given that three interceptors were launched. We record the various estimates and bounds for this excursion in Table 1-3 which is comparable to Table 1-2 above.

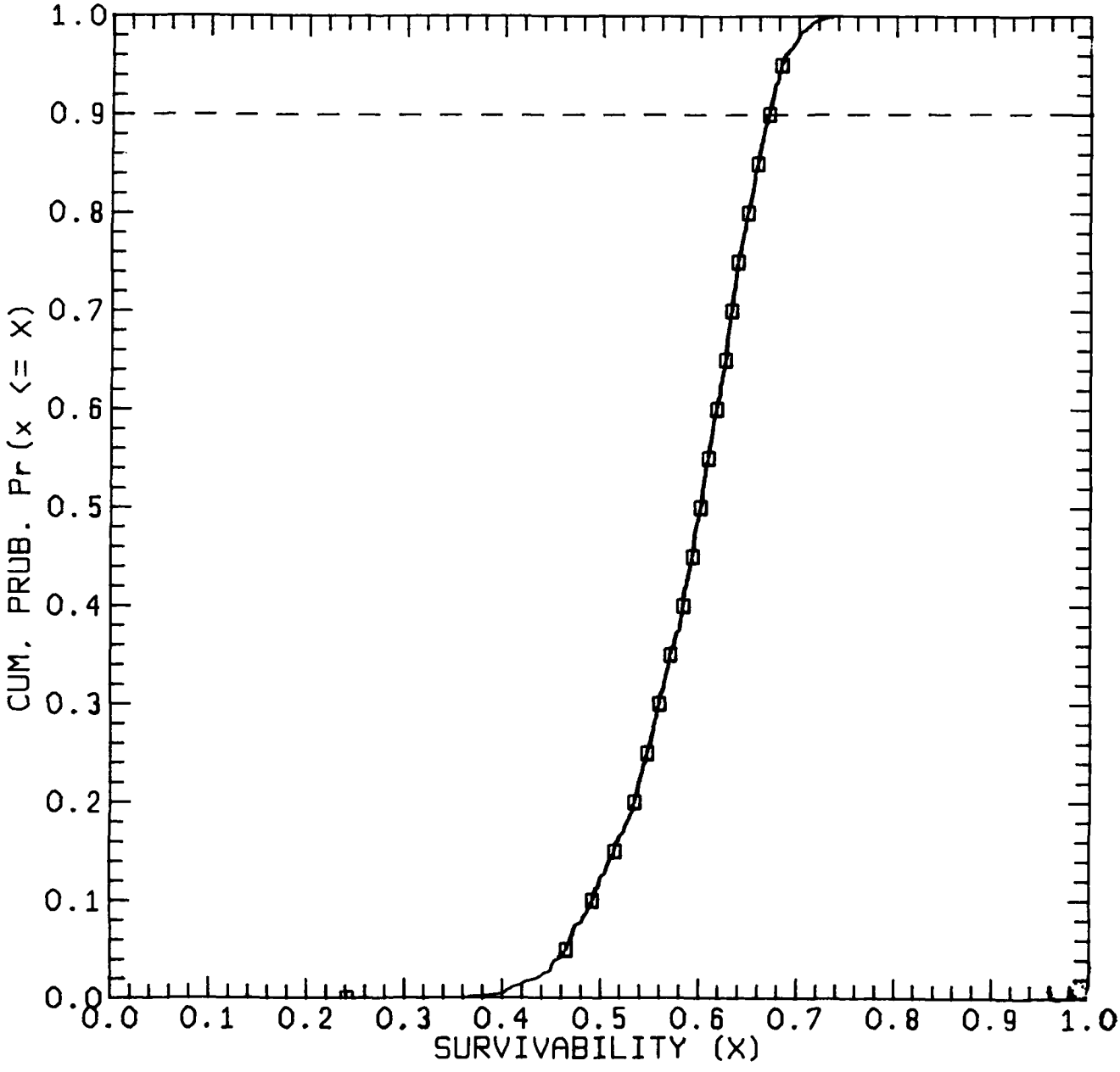

Figure 1-19. Example early NMD survivability distribution with uncertainties reduced - single interceptor launched.

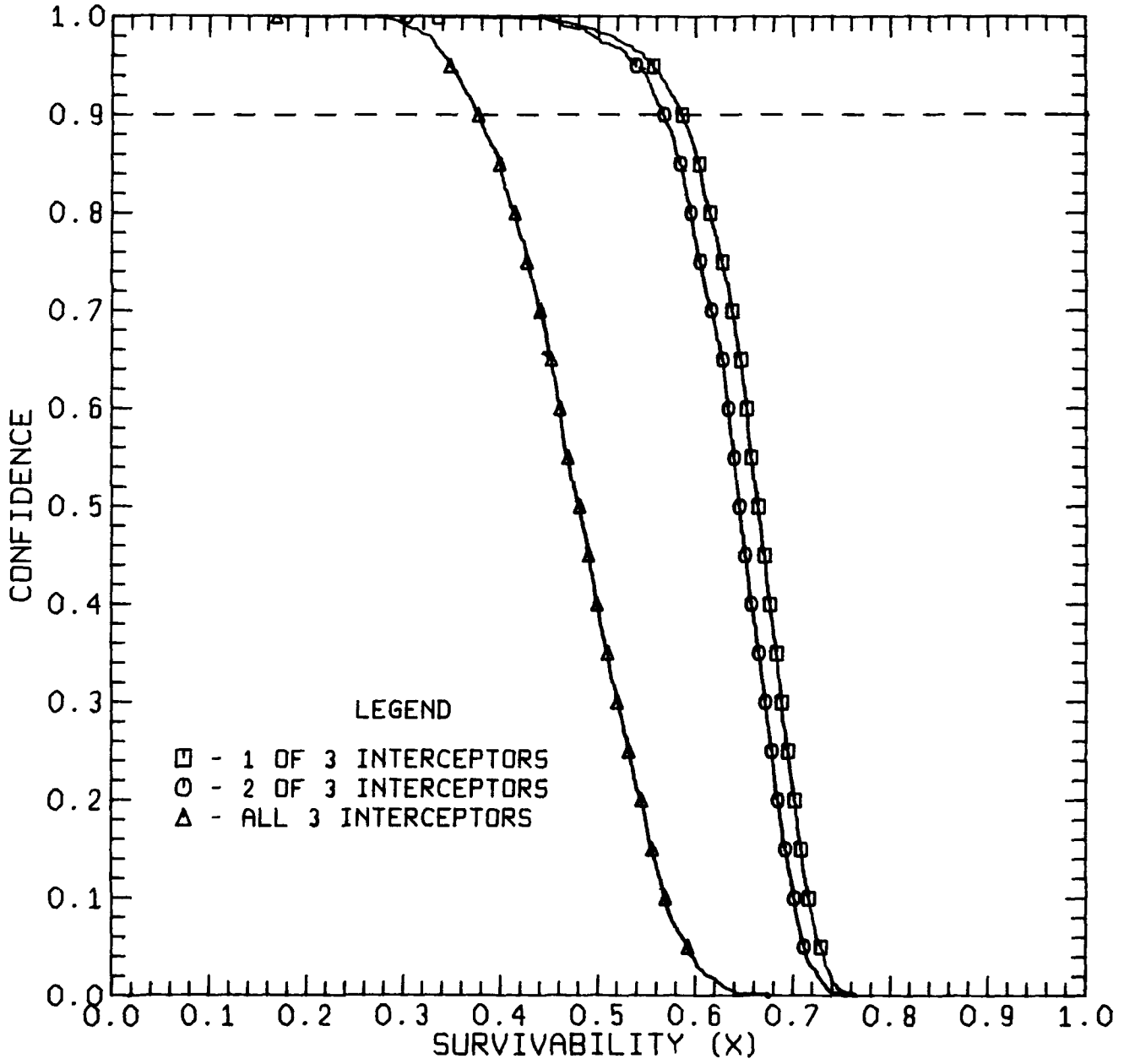

Figure 1-20. Example early NMD confidence functions with uncertainties reduced - three interceptors launched.

Table 1-3. Example early NMD survivability estimates and bounds with uncertainties reduced.

Case	Mean	S.D.	90% LCB	50% LCB
1 of 1	0.590	0.07	0.49	0.60
1 of 2	0.650	0.05	0.58	0.65
2 of 2	0.529	0.07	0.43	0.53
1 of 3	0.656	0.05	0.58	0.67
2 of 3	0.638	0.06	0.57	0.66
3 of 3	0.477	0.07	0.38	0.48

Note that for all practical purposes the means of the survivability distributions have not changed, (any variation is due to sampling differences) but that the sample standard deviations are consistently smaller than before, only half as large in some cases. Because of the reduction in uncertainty the 90% LCBs on system-level survivability are also consistently higher than before, but there is essentially no change in the 50% LCBs. Having the system model and the capability to repeatedly simulate system performance permits many excursions to determine optimal ways to improve system survivability and reduce its variability.

This example has presented the methodology that we propose be employed to develop system-level survivability distributions and quantitative estimates with associated confidence from similar information for subsystems, components, and piece parts. The Monte Carlo simulation method is completely general, is applicable at any level of complexity, and produces legitimate results - that is, there are no necessary simplifying assumptions or truncations.

There are, however, two major difficulties in applying this method for a complex system. The first is developing legitimate survivability distributions for the constituent elements of a system. This must be done from test data, from simulations using validated codes that estimate damage and determine survivability, and by using judgment about the effects upon components for the stressing scenarios of interest. These survivability distributions for system elements will include the effects of both systematic uncertainties and random errors.

The second major difficulty in applying the methodology is the present difficulty in developing the models and performing the simulation. This second difficulty can be overcome by developing generic software tools to accommodate this type of analysis. We believe that this distributional approach to system survivability has been made viable by orders of magnitude improvement in the speed and cost of computer calculations over the past decade.

SECTION 2

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APPENDIX A

PROBABILITY TUTORIAL

A.1 PRELIMINARY CONCEPTS.

The remainder of this report is devoted to a summary of some of the important concepts of probability and statistics which are pertinent to the analysis of those systems of concern here. Much of the material is of an elementary nature, but is presented to provide a common vocabulary and understanding among readers, most of whom can safely skim quite rapidly through the bulk of the text. We will avoid the abstruse aspects of the material as much as possible so that few, if any, readers will be either baffled or intimidated.

A.1.1 Systems And Experiments.

The basic object of interest is a system by which we mean a collection of "things" (the components of the system). A system is involved in some activity (experiment or action) which produces one or more results (outcomes). An exhaustive and mutually exclusive list of outcomes is called an outcome list or a sample space. For example, the experiment of tossing a coin involves a system made up of a coin, a person to toss the coin, and a surface on which the coin lands; the outcomes of interest are whether the coin lands with "heads" or "tails" showing, the possible result of the coin landing on its edge normally being ignored.

A component of a system may itself be another system, in which case it may be referred to as a subsystem. In this manner a system may contain several nested layers of subsystems. In the coin-tossing experiment, the tosser (the person) is a system made up of several biological subsystems (nervous system, digestive system, etc.), each of which is made up of other systems (brain, nerves, receptors, etc.)

A.1.2 Outcomes And Events.

An experimental event is a collection of one or more possible outcomes of the experiment. If we expand the coin-tossing experiment by simultaneously tossing two coins -- say, a nickel and a dime -- then we could define a sample space consisting of the four outcomes: "both heads," "both tails," "a nickel head and a dime tail," and "a nickel tail and a dime head." The outcomes in a sample space are referred to as "two heads," "at least one head," etc. Of course, the three events "no heads," "one head," and "two heads" also constitute a sample space because they are exhaustive (any experiment will have one of these outcomes) and they are mutually exclusive (only one of them can occur for a given trial of the experiment).

A set of outcomes of an experiment may be classified as being either discrete or continuous depending upon the range of values which can be taken. A precise definition of these terms would take us far afield, but generally speaking an outcome set is discrete if it consists of separated entities such as "head" and "tail" or "1," "2," and "3." Any outcome set containing a finite number of members is discrete, but sets with an infinite number of members -- such as the number of tosses of a single coin before a head occurs -- are possible. A continuous set is essentially one in which the outcomes can conceptually be measured to any degree of precision. The continuous sets of interest to us are usually numerical intervals such as the set of

all real numbers between zero and one. Every continuous set contains an infinite number of members.

We will speak of an outcome as being discrete (continuous) if it is a member of a set which is discrete (continuous).

A.1.3 System Structure.

It is occasionally useful to classify a system according to its structural complexity:

- Series:
  A system whose components are operationally connected in series, such as a battery connected to a light bulb.

- Parallel:
  A system whose components are connected in parallel, such as two batteries hooked together in parallel so that either or both of the batteries will supply power.

- Series/Parallel:
  A combination of series and parallel subsystems, such as two batteries in parallel connected to a light bulb.

- Complex:
  None of the above, such as two batteries and two light bulbs hooked together so that either battery can power either light bulb.

A.1.4 System Models.

Our ultimate interest in a real-world system is how it actually performs. Systems are sometimes available for experimental use and can be exercised as much as we wish -- there is essentially no limit to how many times we can toss a penny. However, most are not so accommodating: some do not even exist (a 1999 Toyota Celica or Columbus' Santa Maria); some cost too much are are too dangerous to experiment with (a space shuttle or a nuclear power plant); some self destruct when operated (a nuclear warhead or a firecracker). To investigate and analyze the behavior of one of these systems we are usually forced to resort to a model of the system rather than the real thing.

Models come in numerous forms and share only the property of being a more or less accurate representation of some of the properties of the actual system. The only model which is 100% accurate and complete is the system itself, so results obtained from any other model must be viewed with some skepticism and used with caution.

The selection of which system properties to include in a model depends primarily upon the questions to be answered, but the availability of tools and materials for both building and analyzing the model must also be considered.

As an example consider an automobile theft-prevention system which consists of a battery, two sensors (either of which is able to trigger an alarm), and the alarm device itself. What are some possible models?

- The sentence above that described the system is a verbal model and is quite adequate for some purposes.

- We might use a camera or an artist to produce a pictorial model (useful for advertising).

- A full-scale mock-up might be useful to make sure the system will fit into the automobile.

- A schematic diagram showing how the components are connected together is needed to indicate how the individual parts interact.

- A wiring diagram to show how the system is hooked up to the car.

- A logic model to analyze the operational logic to make sure that if all of the components work properly, the system will perform as desired and to investigate the consequences if one or more of the components fail. Logic models can be constructed in many different ways; some possibilities include:

  - Verbal:
    "If the battery (B) is good and either or both of the sensors (S1 and S2) operates properly and the alarm (A) is good, then the system is good,"

  - Symbolic (mathematical):
    S = B and (S1 or S2) and A,

  - Event Table (1 = "good," 0 = "bad") (see Table A-1),

  - Event Tree (This is simply a graphical representation of the event table; it is a convenient tool with which to work when we attach probabilities to the events) (see Figure A-1),

  - Fault Tree (this shows the ways in which the system can fail) (see Figure A-2).

A.2 PROBABILITY.

The operation of most systems involves uncertainty. Thus, when we toss a con, we are certain that it will come up either heads or tails, but are not certain which. Probability and statistics are the mathematical sciences which attempt to define, measure, and manipulate numerical measures of certainty or uncertainty.

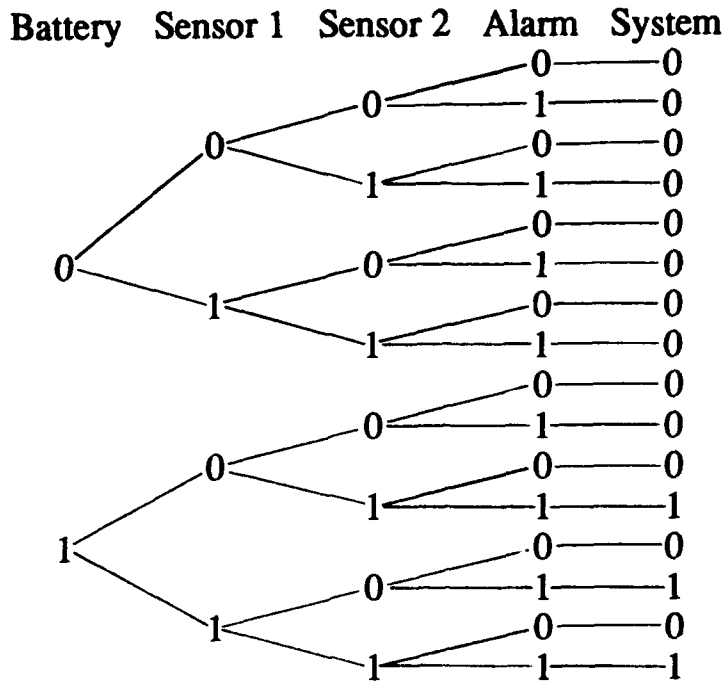
The probability of an outcome of an experiment is a measure of the certainty with which that outcome will occur when the experiment is performed -- that is, when the system operates. The measure is made on a scale from zero to one where a value of one means that the outcome is sure to occur and a measure of zero that it never occurs (or, as we will see later, can be expected never to occur). The sum of the probabilities of all members of a sample space is always unity.

It is fairly common to define the probability of an outcome as the relative frequency (proportion of number of trials) or occurrence of that outcome in an infinite number of trials. This is not a satisfactory operational definition because of the infinite-number-of-trials requirement. Nevertheless, the statement suggests that the observed relative frequency in a large number of trials should be at least close to THE probability. Use of the mathematical theory of probability bears this out and

Table A-1. Automobile theft prevention system event table.

Battery	Sensor 1	Sensor 2	Alarm	System
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	0
1	1	1	1	1

also provides some help in deciding what "large" and "close to" mean (we will see some of this later on in this paper).



o Fault Tree (this shows the ways in which the system can fail):

Figure A-1. Automobile theft prevention system event tree.

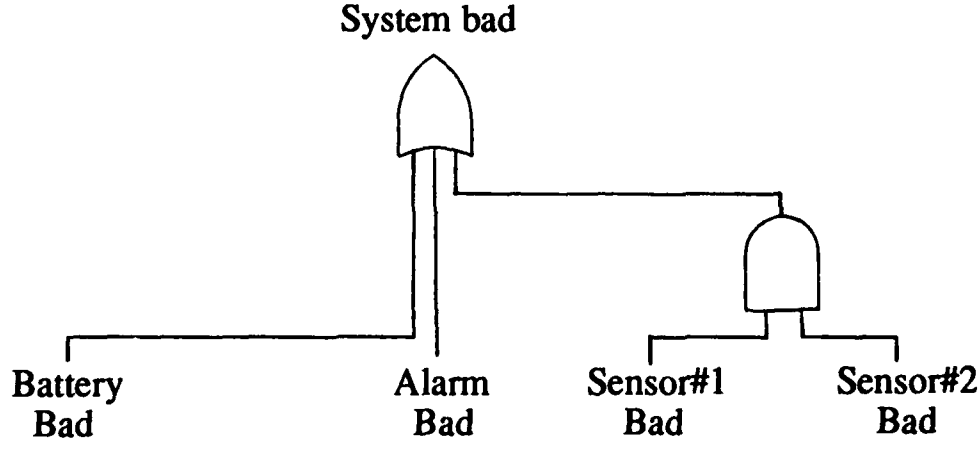


Figure A-2. Automobile theft prevention system fault tree.

A.2.1 Obtaining Probabilities.

There are several ways in which outcome probabilities are obtained:

- Assumption:
  This is used in all theoretical studies. We simply say "if the probabilities of a set of outcomes are such-and-such, then . . . " In this manner the large, indispensable, and sometimes impenetrable body of probability theory has been established.

- Symmetry:
  Because the single-coin-tossing experiment has two outcomes and there does not appear to be any reason why one should occur more frequently than the other, we are usually willing to say that the probability of each is 1/2. However, a little thought suggests that the uncertainties here lie more in the tosser than the coin. In fact, it seems quite likely that a tossing machine could be built that would always produce only one outcome, so the blithe assumption of symmetry may be far from correct.

- Intuition (guessing guided by experience and/or education):
  What is the probability that a tossed irregular polyhedron will land on a certain face? What is the probability that you will die of brain cancer at the age of 73? What is the probability that a stick of dynamite will explode when hit with a sledge hammer?

Such probabilities as these may be extremely difficult to obtain, but frequently we are in a situation that requires some kind of an answer. One way out is to hedge a guess by saying, for example, "The answer is probably between 0.3 and 0.5." This of course introduces another unknown probability, but it seems to narrow things down a bit, and if this statement comes from an "expert," we are much more likely to accept it with a reasonable amount of confidence. And, if the expert is willing to define "probably" as "about 0.95," we may even feel pretty good about it! This approach is closely tied to Bayesian probability which will be discussed later in more detail.

- System Testing:
  Repeated trials of a system's operation is the only way to be sure that we are sneaking up on the real probabilities of the system's outcomes. If for some reason a sufficiently large number of trials is impossible, we are forced to turn to some other approach.

- System Model Testing:
  When a system is relatively complex, it may be possible to obtain or estimate probabilities associated with the system's components and then test a system model involving the components to estimate system probabilities. However, when this method is used, it is vital to remember that the results obtained apply to the model itself and may or may not accurately represent the corresponding features of the modeled system.

A.2.2 Random Variables And Probability Distributions.

When an experiment is performed, one and only one of the outcomes in the sample space will occur. We are frequently concerned with how the total probability

(1.0) is distributed over the various outcomes -- that is, what is the probability of each possible outcome?

If there are a finite number of outcomes, a simple tabulation of the outcomes and their probabilities is all that needs to be done. However, if the outcome set is infinite in extent, a complete tabulation is impossible and some other device must be used. This device is a random variable which is simply a variable whose value ranges over some set of real numbers where each member of that set is associated with one or possibly several outcomes (but no outcome is associated with more than one of the real numbers). A random variable is called discrete if it is defined on a discrete set of outcomes and continuous if the set is continuous.

For example, if we toss six coins simultaneously, we might associate with each possible outcome (such as "HHTHTH") the number of heads (4). The value range of the discrete random variable "NUMBER OF HEADS" is then the integer set {0,1,2,3,4,5,6}. It is common practice to use an uppercase letter as the symbol for a random variable. Thus, if we let N = "NUMBER OF HEADS", the question "What is the probability of five heads?" can be written symbolically

$$Pr(N = 5) = ?$$

where "Pr" means "probability."

We now define two probability functions of a discrete random variable. The first is the Probability Mass Function (PMF) which is defined by

$$PMF_A(x) = Pr(A = x)$$

where x is a numerical variable on the value range of the random variable A.

The second function is the Cumulative Distribution Function (CDF) defined by

$$CDF_A(x) = Pr(A \leq x)$$

where again the domain of x is the value range of A. Note that because a probability is always non-negative, a CDF is a non-decreasing function.

The subscript on PMF and CDF is omitted if only one random variable is under discussion or it is otherwise obvious which random variable is being considered.

Example A.2.2a:

A fair coin is tossed once. (The adjective "fair" is customarily used when we invoke a symmetry definition of probability; in this case that means that we assume that the probability of a head is the same as that for a tail and, consequently, each must be 0.5.) Let N = "number of heads." The value range for N is simply the set {0,1} and we can readily tabulate the PMF and the CDF (see Table A-2) (any symbol can be used for the dummy argument of the function; we use "x" in this example and will use other symbols in other examples as the whim strikes us):

Note that the PMF entries must always sum to unity.

Table A-2. Fair coin example PMF and CDF.

x	PMF(x)	CDF(x)
0	0.5	0.5
1	0.5	1.0
	1.0	

Example A.2.2b:

Two fair coins are tossed. Let B = "number of heads." By symmetry there are four equally likely outcomes which, letting the first letter indicate the result of one coin and the second letter the result of the other, are HH, HT, TH, and TT. Note that the second and third outcomes give the same number of heads (1) (see Table A-3).

Table A-3. Two fair coins example PMF and CDF.

y	PMF(y)	CDF(y)
0	0.25	0.25
1	0.50	0.75
2	0.25	1.00
	1.00	

Example A.2.2c:

A fair coin is tossed until a head occurs. Let C = "number of such tosses." Here the value range of C is the infinite set of all positive integers. A complete table cannot be constructed, but a partial one may be useful (Table A-4). Noting that the event "C = x" requires x-1 tails followed by a head and assuming independence of tosses (see below), the formula for the PMF, where t is the number of tosses, is $PMF(t) = (0.5)^t$, t = 1,2,3,. . .

Being a bit clever (or using a mathematical handbook) the formula for the CDF can be found to be $CDF(t) = 1 - (0.5)^t$. Thus $CDF(10) = 1 - (0.5)^{10} = 1 - 1/1024 = 0.9990$. The probability of C being greater than t is 1 - CDF(t), so the probability of needing more than 10 tosses to get a head is very small -- about 0.001.

Example A.2.2d:

Consider a system that consists of a piece of cardboard on which a circle is drawn and a pointed spinner is mounted at the center of the circle in such a manner that it can freely revolve when flicked with a finger. The circle is divided into five equal segments marked 2,3,5,7, and 11. Similar devices frequently appear with board games made for the young at heart, and a roulette wheel or a Wheel of Fortune are

Table A-4. Tossing fair coin until head occurs example PMF and CDF.

t	PMF(t)	CDF(t)
1	0.5	0.5
2	0.25	0.75
3	0.125	0.875
4	0.0625	0.9375
•	•	•
•	•	•
•	•	•
	1.0000	

advanced versions of the same idea (these two have little fences to prevent the spinner from landing on a boundary -- an endless source of childhood arguments). The experiment consists of flicking the spinner and noting the segment where the spinner point is located when the spinner stops. Let the random variable D be the number associated with that segment.

Using symmetry, we will assume that the pointer has no preference for any particular stopping point, and, consequently, each of the equal segments has a probability of 0.2 of containing a stopping point. Table A-5 gives a tabulation of the PMF and CDF for this example.

Table A-5. Spinner example PMF and CDF.

s	PMF(s)	CDF(s)
2	0.2	0.2
3	0.2	0.4
5	0.2	0.6
7	0.2	0.8
11	0.2	1.0
	1.0	

It is clear that we can divide the circle in Example A.2.2.d in any manner we choose -- that is, we can have as many segments as we wish and they can be of

varying lengths if desired. Then, assuming a fair spinner, the probability associated with any segment is just the proportion of the circumference occupied by that segment. This is turn says that the probability behavior of any system with a discrete set of outcomes can be, at least conceptually, modeled by a spinner system. There are some practical difficulties involved if the number of outcomes is infinite as in Example A.2.2.c, but in that case we might lump all of the outcomes with a value above, say, 10 into a single segment with -- as we have seen -- a probability of about 0.001 and have a good, but not perfect, model.

With this in mind, it is a small, but very significant, step to consider what happens if we replace the segmented scale on the spinner with a continuous one -- for example, a uniform scale from 0 to 10 (the 0 and 10 being located at the same point).

First, for example, we might ask "What is the probability that the spinner will stop at exactly 2.5?" To expedite matters, let the value of the continuous random variable S be the number on the spinner scale where the spinner stops. Then the question becomes

$$\Pr(S = 2.5) = ?$$

By symmetry we must agree that each point on the scale -- and hence each real number between 0 and 10 -- is equally likely to occur; but as there are an infinite number of such points, the answer must be $1/\infty = 0$. On the other hand some number will occur for each spin, so we are now faced with the situation -- noted earlier -- where an outcome with zero probability can indeed happen. In any event there is no hope of finding a PMF for any continuous system.

However, if we change our question to

$$\Pr(S \leq 2.5) = ?$$

We are talking about an interval rather than a point. In this case the interval is the set of all numbers between 0 and 2.5, and because that is one quarter of the scale, the answer is 0.25. Consequently, although we are unable to find a PMF, we can find a CDF! In fact, a little thought tells us that for this spinner the CDF formula is

$$\text{CDF}(x) = x/10, \quad 0 < x \leq 10.$$

The careful reader will have noted that we have given the point where 0 and 10 coincide the value of 10 -- that is, 0 is not in the value range but 10 is. The fact that a single point has a zero probability of occurring indicates that the other way around would be perfectly satisfactory, but this is the conventional way to do it.

The graph of this CDF is just a straight line segment going from the origin (0,0) to the point (10,1), and -- like any CDF -- is a non-negative, non-decreasing function. So, putting our calculus glasses on, we observe that this CDF could be obtained by integrating some non-negative function whose domain includes the interval [0,1] -- that is, there is some function f such that

$$\int_0^x f(t)dt = x/10, \quad 0 \leq x \leq 10.$$

But the Fundamental Theorem of Calculus says that if the integral in this equation is differentiated, the result is simply f(x). So if we differentiate both sides of the above equation, we get

$$f(x) = d/dx\,(x/10) = 1/10$$

whose graph is a horizontal line segment.

This function is called the Probability Density Function of the random variable (S in this example) and is denoted by $PDF_S(x)$ -- that is,

$$PDF_S(x) = 1/10, \quad 0 < x \leq 10.$$

Note that, for example, we can find $Pr(3 \leq S \leq 7)$ by subtracting $Pr(S \leq 3)$ from $Pr(S \leq 7)$. Generally,

$$\begin{aligned} Pr(a \leq S \leq b) &= Pr(S \leq b) - Pr(S \leq a) \\ &= CDF_S(b) - CDF_S(a) \\ &= \int_a^b PDF_S(t)dt \\ &= \text{area under graph of } PDF_S \text{ between a and b} \end{aligned}$$

We may replace the uniform scale on the circle of our spinner board by any scale we wish (the physical length remains the same of course): shorter (1 to 4), longer (-1000 to 1000), or -- more interestingly -- non-uniform (think of writing a scale on a rubber band and then stretching or compressing the band in various ways as long as the total length remains fixed. For example, if we took our scale from 0 to 10, stretched the middle and compressed the ends, we would increase the probability of obtaining numbers near 5 and decrease it for numbers near 0 and 10; the graph of the PDF would then be humped up in the middle and might (or might not) go to zero at the ends -- in any event the area under the curve would remain unity.

By stretching our minds as well as the rubber band, the scale can include infinity (∞) on either or both of the ends of the scale. With this provision, a spinner can be constructed to model the probability of any random variable.

The term random sample will be encountered frequently in this paper. A random sample of size n is simply a set of n observed values of a random variable. For example, if we have a spinner board for the random variable, the numbers obtained by flicking the spinner n times make up a random sample.

There are different notations (and sometimes different names) in common use for the various probability functions. In particular -- and because mathematicians have not been accustomed to the use of a multiple-letter name for a variable -- the cumulative distribution function is frequently symbolized by a single upper case letter and the corresponding density or mass function by the same letter in lower case (the random variable involved may or may not be attached as a subscript).

A.2.3 Distribution Measures.

There are many numerical measures associated with a random variable which provide information about the nature of its probability distribution.

The most important of these are defined as expected values of certain functions of the random variable. An expected value is simply a weighted average of all of the possible values of such a function, where the weight applied to a function value is the probability (for a discrete random variable) or the density (for a continuous one) associated with the value of the random variable. Let R be a random variable and denote its value range by VR. Let E(f(R)) denote the expected value of the function f(R). Then, for discrete R, the expected value of f(R) is

$$E(f(R)) = \sum_{t} f(t) \times PMF_R(t)$$

and for continuous R, the expected value is

$$E(f(R)) = \int_{t} f(t) \times PDF_R(t)\, dt.$$

If f(R) = R, the expected value is known as the mean of R. It is a measure of the average value of R or the center of the distribution and is symbolized by E(R) or μ_R. Other common measures of the average include the median (the value for which the CDF = 0.5) and the mode (the value with the greatest probability or density).

If $f(R) = (R - \mu_R)^2$, the expected value is called the variance of R and is a measure of variability or spread or dispersion of the distribution. It is symbolized by σ_R^2 or Var(R). The positive square root of the variance is called the standard deviation and is symbolized by σ_R or SD(R). Another common measure of the variability is the range (the difference between the highest and lowest values of R).

A.2.4 Distribution Parameters.

We have seen that the formula

$$PDF_R(t) = 1/10, \quad 0 < t \leq 10,$$

defines a uniformly distributed random variable on the interval (0,10). If we replace the "10" by the symbol "b", the resulting formula

$$PDF_R(t) = 1/b, \quad 0 < t \leq b$$

$$b > 0$$

defines a class of uniformly distributed random variables, and b is called a parameter of the distribution. Note that the allowable range for a parameter should be explicitly stated.

In many practical problems we make the assumption that a distribution belongs to a certain class which is defined by one or more parameters and then attempt to find or estimate the values of the parameters and hence obtain the specific distribution associated with our specific problem.

In some cases a distribution parameter is also a distribution measure (in the example above the parameter b happens to be the range of R). It is usually possible to give explicit formulas for at least some of the distribution measures in terms of the parameters, but this cannot always be done.

We list below the probability mass or density functions of several commonly encountered random variable distribution classes and the formulas for the mean and variance of the random variable. In all cases R is the random variable name, and the parameters are indicated by either distribution measure symbols or just lower case letters (the letter "x" will be used as the argument of the PMF or PDF and is not a parameter):

Binomial (discrete):

$$PMF(x) = n!/(x!(n-x)!)p^x(1-p)^{(n-x)},\ x = 0,1,2,\ldots,n$$

$$0 < p < 1;\quad n = 1,2,3,\ldots$$

$$\mu = np;\quad \sigma^2 = np(1-p)$$

For example, R = the number of successes in n trials of an experiment whose two outcomes are "success" and "failure." The other parameter, p, is the probability of a success on a single trial. If n = 1, the resulting distribution is called a Bernoulli distribution -- e.g., tossing a single coin once.

Normal (or Gaussian) (continuous):

$$PDF(x) = (1/(\sigma\sqrt{2\pi}))\exp(-((x-\mu)^2)/2\sigma^2),\quad -\infty < x < \infty$$

This is the most well-studied and frequently used continuous probability distribution, primarily because it is a limiting form of many other distributions and consequently in many situations serves as a good approximation.

The graph of the PDF of a normal distribution is the well-known "bell-shaped" curve which is symmetric about its mean. The standard normal distribution is one whose mean is zero and whose variance (and standard deviation) is unity. This distribution has been extensively tabulated because the probability behavior of any normal random variable can be obtained from such a table.

Beta (continuous):

$$PDF(x) = (a+b+1)!/((a-1)!(b-1)!)\,x^{(a-1)}(1-x)^{(b-1)}$$
$$0 < x < 1$$
$$a > 0;\ b > 0 \text{ (usually both a and b are integers)}$$
$$\mu = a/(a+b);\ \sigma^2 = ab/((a+b)^2(a+b+1))$$

This distribution is most often encountered in Bayesian probability and statistics. Note that if a = b = 1, the distribution becomes the uniform distribution on the unit interval.

Uniform (continuous):

$$PDF(x) = 1/(h-l),\quad l \le x \le h$$

$$\mu = (l+h)/2;\ \sigma^2 = (h-l)^2/12.$$

This is the simplest of all continuous distributions. The graph of its PDF is a horizontal line segment 1/(h-l) units above the x-axis and extending from x = l to x = h. The CDF is a straight line segment joining the points (l,0) and (h,1).

A.2.5 Joint Distributions.

The outcomes of an experiment may involve more than one random variable -- for example, if we select a member of Congress at random, we could define the random variable H = height of the person selected and W = his or her weight (and many others if we wished). We could study the behavior of each random variable separately by simply ignoring the others, or we might be interested in their joint behavior.

To illustrate the ideas involved with joint distributions let an experiment be tossing a fair coin three times. Letting H denote a head and T a tail and writing down the three results in sequence, we see that there are eight possible outcomes, each (by symmetry) having a probability of 1/8. Define two random variables: N = number of heads (0,1,2, or 3) and R = the length of the longest run, where a run is a set of successive flips having the same face (1,2,3). Table A-6 lists the possible outcomes, their probabilities, and the associated values of N and R.

Table A-6. Outcomes and probabilities for joint distribution of N & R.

Outcome	Probability	N	R
hhh	1/8	3	3
hht	1/8	2	2
hth	1/8	2	1
htt	1/8	1	2
thh	1/8	2	2
tht	1/8	1	1
tth	1/8	1	2
ttt	1/8	0	3
	1.0		

Using this table we can immediately construct the individual PMFs for N and R (Tables A-7 and A-8) and the joint PMF of N and R together (Table A-9).

The table entries are the joint probabilities of N and R and the notation $PMF_{N,R}$ is defined to be

$$PMF_{N,R} = Pr(N = n \text{ and } R = r).$$

Table A-7. PMF_N.

n	Pr(N = n)
0	1/8
1	3/8
2	3/8
3	1/8
	1.0

Table A-8. PMF_R.

r	Pr(R = r)
1	2/8
2	4/8
3	2/8
	1.0

Table A-9. Joint $PMF_{N,R}$.

		r		
		1	2	3
n	0	0	0	1/8
	1	1/8	2/8	0
	2	1/8	2/8	0
	3	0	0	1/8

Note that if the entries are summed across each row and then down each column, the results are identical to the individual PMFs of N and R; when they are obtainable in this manner, these PMFs are referred to as marginal distributions.

A.2.6 Conditional Distributions.

There are situations where we are interested in only some of the outcomes of an experiment rather than all of them. The set of these outcomes is called the conditioning set or event. We then simply ignore those outcomes which are not in the conditioning set and go on from there. For example, in the 3-coin experiment

just discussed assume we are interested only in those outcomes which contain exactly one head. Looking at Table A-6, the members of the conditioning set are seen to be htt, tht, and tth. These are still equally likely, and so here each has a probability of 1/3. Now, what is the probability distribution of the number of runs given that the outcome is in the conditioning set? We can symbolize this question by

$$\Pr(R = r \mid N = 1) = ?$$

where the vertical bar separating the event of interest from the conditioning event is read "given." Looking at the conditioning set and remembering each outcome there has a probability of 1/3, the answers to the question are:

$$\begin{aligned}\Pr(R = 1 \mid N = 1) &= 1/3,\\ \Pr(R = 2 \mid N = 1) &= 2/3,\\ \Pr(R = 3 \mid N = 1) &= 0.\end{aligned}$$

which defines the required conditional probability mass function.

The conditional PMF can be obtained directly from the unconditional joint PMF of N and R (Table A-9) rather than the table of outcomes (Table A-6). We first observe that a conditioning event shrinks the outcome set to just that event (in our example to the row for n = 1). The sum of the original probabilities in the conditioning event is 1/8 + 2/8 + 0 = 3/8 [which of course is Pr(N = 1)]. However, in the conditional distribution the sum of the probabilities must equal 1. The most reasonable way of doing this is to multiply each of the unconditional probabilities by some constant k so that the new sum is unity. Therefore, we want to find k so that

$$(1/8)k + (2/8)k + (0)k = 1$$

which says

$$\begin{aligned}k &= 1/(1/8 + 2/8 + 0)\\ &= 1/(3/8)\\ &= 1/\Pr(N = 1).\end{aligned}$$

In short,

$$\Pr(R = r \mid N = 1) = \Pr(R = r \text{ and } N = 1) / \Pr(N = 1).$$

A.2.7 Statistical Independence.

The basic idea of statistical independence [usually just ("independence"])] between two outcomes is that they do not influence each other. Thus, when tossing a coin twice, what happens on the first toss presumably does not influence what happens on the second, and so we are inclined to postulate independence between the outcomes of the two different tosses. More precisely, and using the concept of conditional probability, the two random variables S and T are defined to be independent if and only if

$$\Pr(S = s \mid T = t) = \Pr(S = s)$$

for all possible values of s and t.

An immediate consequence of this definition is the theorem that S and T are independent if and only if

$$Pr(S = s \text{ and } T = t) = Pr(S = s) \times Pr(T = t)$$

for all possible values of s and t. (This is sometimes used as the definition , in which case our definition becomes a theorem.) The theorem can be extended to more than two random variables.

In the 3-coin example we have found that $Pr(R = 1) = 2/8$ and $Pr(R = 1|N = 1) = 1/3$, so by the definition, R and N are not independent (and are said to be dependent). The theorem could also be used by noting that $Pr(R = 1 \text{ and } N = 1) = 1/8$ which does not equal $Pr(R = 1) \times Pr(N = 1) = 2/8 \times 3/8 = 6/64$.

The theorem in effect says that R and N are independent if and only if each entry in the joint PMF table is equal to the product of the row and column totals associated with the entry (those totals of course define the individual PMFs for R and N).

The most important use of independence in both theory and practice is that it allows us to calculate the probability of the joint behavior of two or more independent random variables by using just the individual distributions. It is very difficult to prove that independence does exist, but, in the interests of simplicity or because of the paucity of data, it is always tempting to postulate that it does. Caution is urged!

All of the ideas discussed in this and the earlier two sections extend to continuous as well as discrete random variables by replacing PMFs with PDFs and summations with integrals.

A.2.8 Functions of Random Variables.

If we know something about a random variable T, can we say anything about a random variable S which is a mathematical function of T such as $S = 3T + 4$ or $S = \sin[\log(T)]$? The answer is a resounding "yes" in many cases of interest and a tentative "maybe" in others. We state below a few of the cases of particular importance which will be used later on. The proofs of these assertions are generally straightforward, but in the interests of brevity will be omitted.

- If S is a linear function of T ($S = aT + b$ for any a and b), then $\mu_S = a\mu_T + b$ and $\sigma_S^2 = a^2\sigma_T^2$.

- If $R = aU + bV$, then $\mu_R = a\mu_U + b\mu_V$.
  If in addition U and V are independent, then

  $$\sigma_R^2 = a^2\sigma_U^2 + b^2\sigma_V^2$$

  and if U and V are each normally distributed, R will be also.

A useful verbal statement of the case where $a = b = 1$ is: if U and V are independent random variables, then the mean of their sum is the sum of their means and the variance of their sum is the sum of their variances.

- When doing computer simulations which involve random variables it is usually necessary to select a random sample from some given probability distribution. If the distribution is continuous, we can select a value of such a random variable as follows:

Let F_T denote the CDF of the distribution from which we wish to sample -- that is, $F_T(x) = Pr(T \leq x)$.

Let F_T' be the inverse of F_T -- that is, if $y = F_T(x)$, then $x = F_T'(y)$.

Let U be a uniformly distributed random variable on the unit interval ($PDF_U(x) = 1$, $0 \leq x \leq 1$). Most computer programming languages have a random number generator which samples from such a distribution.

Let W be a random variable such that $W = F_T'(U)$.

Then W has the same probability distribution as T.

The only problem with this procedure is finding the inverse function, but adequate approximations have been developed for all common distributions.

- The Central Limit Theorem says that the CDF of the sum of the n members of a random sample (n independent and identically distributed random variables) approaches the CDF of a normal distribution as n gets larger regardless of the CDF of the parent distribution of the random sample. This theorem is extremely useful because it permits us to get good answers to many questions by using a normal as the approximating distribution.

A.2.9 Bayesian Probability.

A Bayesian is a person who believes that any quantity whose value he does not know is a random variable and that it is possible to express his current knowledge about such a random variable in the form of a PDF or PMF. Then, as additional knowledge becomes available he alters the distribution to accommodate the new information.

There has been much discussion, debate, and dissension between Bayesians and Classicists (non-Bayesians) over years, but the Bayesian philosophy has established itself as a respectable approach to many problems. A. W. Drake said in his 1967 textbook Fundamentals of Applied Probability Theory:

"The Bayesian approach represents a significant departure from the more conservative classical techniques of statistical analysis. Classical techniques are often particularly appropriate for purely scientific investigations and for matters involving large samples. Classical procedures attempt to require the least severe possible assumptive structure on the part of the analyst. Bayesian analysis involves a more specific assumptive structure and is often described as being decision-oriented. Some of the most productive applications of the Bayesian approach are found in situations where prior convictions and a relatively small amount of experimentation must be combined in a rational manner to make decisions among alternative future courses of action."

That statement strongly suggests that when dealing with large-scale systems, which of necessity are based on scanty amounts of firm data, a Bayesian posture is indicated. We agree with this viewpoint and will use it freely in later sections.

A.3 PARAMETER ESTIMATION.

One of the major problems of statistical inference is that of estimating the value of some random variable measure, the mean and variance being of particular interest because taken together they usually provide a good idea of the overall probabilistic behavior of the random variable. Both a point estimate (a single value estimate) and an interval estimate which has some measure of certainty associated with it will be discussed in the following sections.

A.3.1 The Blood Cholesterol Example.

Assume that I wish to determine my blood cholesterol level (BCL). The laboratory where I go has a testing machine, but it has been found to produce somewhat erratic results. In light of this variability how can I obtain a reasonably good estimate of my BCL?

Let the result of a test made by the machine be denoted by the random variable C with mean μ_M and variance σ_M^2. The machine produces answers to only three decimal places which says that C is a discrete random variable, but it seems quite reasonable to assume that C is continuous as this will greatly simplify both the theoretical and computational aspects of the problem.

The more or less obvious approach to get a point estimate of BCL is to have the test repeated several times and then calculate an average of the resulting values. There are numerous averages one can use, but the sample mean (the sum of the values divided by the number of them) is the most commonly used one and can be shown to have many good statistical properties.

Most people would agree that the larger n is, the better the final estimate will be -- that is, the closer it will probably be to the real BCL value. To investigate this matter we need to consider what might happen if we repeated this experiment many times and looked at the distribution of the resulting sample means. Symbolically, let the random variable M denote the mean of a random sample of size n. What can we say about the probability behavior of M?

M is a linear function of the sample values of C, and it follows that $\mu_M = \mu_C$ and $\sigma_M^2 = \sigma_C^2/n$. Therefore, regardless of the size of n, the mean of M is equal to the mean of C. However the variance of M gets smaller than that of C as n gets larger, and because the variance is a measure of the spread of the distribution, the distribution shrinks down around μ_M. This behavior is known as the Law of Large Numbers and says that our intuition mentioned above was indeed correct.

Our ultimate interest is of course in the value of BCL, and it is pertinent to wonder if μ_M (which is also μ_C) is the same thing as BCL. The answer is that we will assume it is, which really means that we are assuming that the machine has been properly calibrated so that it has no bias. Unless a more accurate, and preferably more precise, measuring device is available to calibrate our machine, any bias would be difficult to detect and impossible to measure.

A.3.2 Probability Intervals.

The Central Limit Theorem discussed earlier says that the CDF of M approaches normality as n gets larger. In fact, it can be shown that the normal approximation gives very good answers for quite modest sample sizes -- say, 20 or even smaller in some cases. So, let us assume that M is actually normally distributed and see what can be said about probability statements concerning M. Let's start with

$$Pr(M \leq x) = ?$$

Recalling the definition of a CDF, our question can be restated as

$$CDF_M(x) = ?$$

Because M is normally distributed (by assumption), we need to evaluate the CDF of a normal random variable. This can be easily done if we change the question into one involving a standard normal random variable Z. The required transformation is

$$Z = (M - \mu_M)/\sigma_M.$$

To see how this works, start with a probability statement for Z:

$$Pr(Z \leq 2.576) = 0.99$$

where the "2.576" and the "0.99" are obtained from a table of the CDF of Z. Making the transformation to M, we get

$$Pr((M - \mu_M)/\sigma_M \leq 2.576) = 0.99$$

which after a little algebraic manipulation and remembering that σ_M is positive becomes

$$Pr(M \leq \mu_M + 2.576\sigma_M) = 0.99.$$

(Instead of the 0.99 we could have chosen any probability and then found in the table a number corresponding to the 2.576 -- for example, 1.96 for 0.975, 1.645 for 0.95, 0.0 for 0.50, and -2.576 for 0.01.)

We see that, although our interest at this time is in μ_M, the event whose probability is 0.99 involves both μ_M and σ_M. To take care of σ_C we will temporarily assume that it equals 10 (that $\sigma_C = 10$ could have been estimated from earlier experience with the machine or with similar machines) and consequently $\sigma_M = 10/\sqrt{n}$. The probability statement now becomes

$$Pr(M \leq \mu_M + 25.76/\sqrt{n}) = 0.99$$

This statement can be read as

$(-\infty, \mu_M + 25.76/\sqrt{n})$ is a 99% probability interval for M.

Because one end of the interval is $-\infty$, it is called a one-sided interval. To get a two-sided interval, we could have started with the statement

$$Pr(-2.576 \leq Z \leq 2.576) = 0.98$$

which says that (-2.576, 2.576) is a 98% two-sided probability interval for Z. Then going through the manipulations used above we would arrive at

$$Pr(\mu_M - 25.76/\sqrt{n} \leq M \leq \mu_M + 25.76/\sqrt{n}) = 0.98$$

which says that (μ_M - 25.76/$\sqrt{n}$, μ_M + 25.76/$\sqrt{n}$) is a 98% probability interval for M.

This and similar statements are useful to study the relationships between the sample size, the precision of the sample mean (one-half of the width of the two-sided probability interval), and the probability associated with that precision. Table A-10 shows a few cases:

Table A-10. Relationship between sample size and the precision of the sample mean.

Sample Size (n)	Precision	Probability
25	5.15	0.98
25	3.29	0.90
100	2.58	0.98
100	1.65	0.90

Our assumption that σ_M = 10 is not required because using the values of a random sample we can estimate σ_M and then use a table of the Student's t distribution instead of the standard normal. There is a different Student's t distribution for each value of n, but it turns out that for sample sizes above 40 or so, the standard normal adequately approximates the Student's t.

A.3.3 Confidence Intervals.

The probability intervals we have constructed tell us where we can expect the sample mean to fall with respect to the unknown distribution mean which is important information when an experiment is being planned. The next question is, "Can these intervals tell us anything about where the unknown distribution mean is after we have actually obtained a random sample and calculated the value of the sample mean?"

Returning to the BCL example and denoting the observed value of a random variable by the corresponding lower case letter, we ask "What can the probability intervals tell us about the location of μ_C after we have obtained m, the observed value of M?" With n = 25 and remembering that $\mu_C = \mu_M$, we have

$$Pr(\mu_C - 5.15 \leq M \leq \mu_C + 5.15) = 0.98$$

which can be interpreted as saying that if we were to obtain many random samples of size n, calculate the sample mean m, then the statement

$$\mu_C - 5.15 \leq m \leq \mu_C + 5.15$$

would be true in about 98% of the cases and false in about 2% of them. The statement can be manipulated to get

$$m - 5.15 \leq \mu_C \leq m + 5.15$$

and it would still be true in about 98% of the cases. Thus, for an observed value of m the interval (m - 5.15, m + 5.15) either does or does not contain μ_C, but because 98% of such intervals would include μ_C, we call the interval a 98% confidence interval for μ_C.

In short, a probability interval is associated with our certainty that before an experiment has occurred an event will happen, and a confidence interval is associated with our certainty that after the experiment has taken place an event did occur.

A.3.4 The Bayesian Viewpoint.

Because a true Bayesian views all unknowns as random variables, he will not accept the analysis we did in obtaining a confidence interval because μ_C was treated as an unknown constant rather than as a random variable. For him one approach would be to assume some probability distribution for BCL. Then working with the joint distribution of BCL and C, he would determine how the results of doing n tests alters the BCL distribution. (The BCL distribution before the experiment is called the prior distribution and the modified one the posterior distribution.) If desired, the posterior distribution could then be used to obtain probability intervals for BCL. (Instead of just probability, one of the adjectives Baye's probability, Baye's confidence, and simply Baye's is frequently used to emphasize the Bayesian nature of the interval.)

Of course, even a Bayesian has to start somewhere. If he knew absolutely nothing about either the machine or the possible values of BCL, he might well simply collect a sample of results and choose some probability distribution which fits the sample in one way or another. This would then become the prior distribution for future experiments or simply used to make statements about BCL which reflect the current state of knowledge.

A.3.5 A Good/Bad Example.

A commonly encountered problem is one in which we need to estimate the probability that a system is "good" (successful, survivable, OK, etc.). The system may be a single component (a coin to be tossed) or a highly complex system (the NMD example discussed in Section 1.3). In any event our interest is in whether, when the system operates, the result is good or bad (not good).

Basically we are dealing with an experiment which has only two outcomes, one of which we will call "good." There is some unknown probability -- say, p -- that a good outcome will occur. Our problem is to estimate p. It is convenient to define a random variable R whose values are 1 if "good" occurs and 0 otherwise. R then has a Bernoulli distribution which, as mentioned in Section A.2.3, is a special case of the binomial distribution with the parameter n equal to 1.

A point estimate of p can be obtained by repeating the experiment a number of times -- say, n -- and counting the number of successes -- say, s. A point estimate of p is then given by the proportion of successes in the sample -- that is, s/n.

To find a classical confidence interval for p we need some information about the probability behavior of S, the random variable whose value is the number of successes obtained in the n trials of the experiment. S has a binomial distribution, so its mean is np and its variance is np(1-p). Because S is discrete, the calculation of exact confidence limits is difficult. Tables and graphs are available for some values of n. Figure A-3 shows such a graph for finding 99% confidence intervals for p; on it we see that if we had 80 successes in 100 trials, a 99% confidence interval for p is (0.63, 0.90). For large n we can invoke the Central Limit Theorem and use a normal approximation of the binomial distribution similar to the way it was done in Section A.3.2.

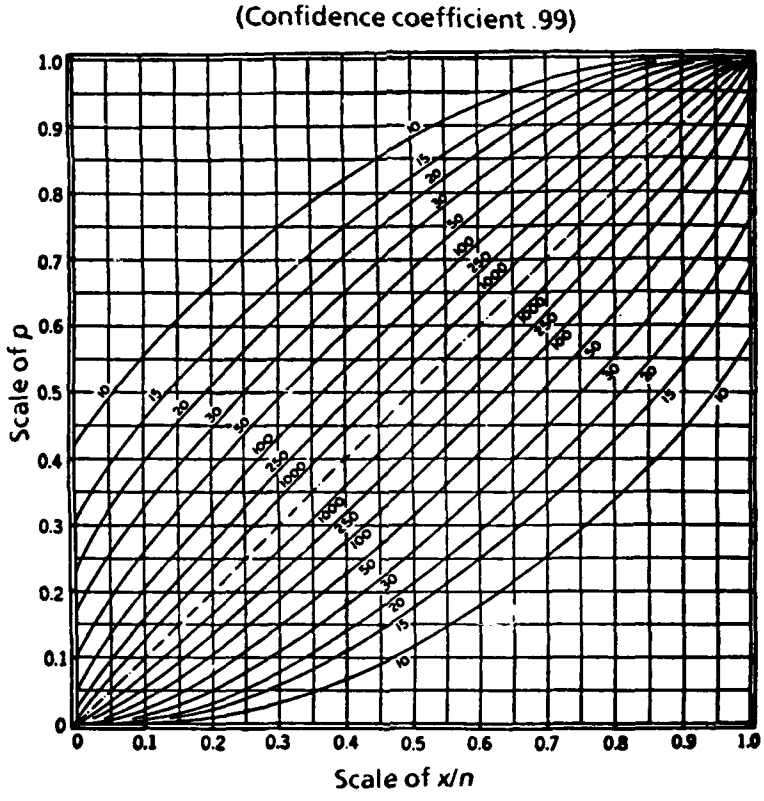


Figure A-3. Confidence limits for proportions.

For a Bayesian approach, the use of Beta distributions is convenient because the range of a Beta random variable is that of a probability -- [0,1] (because our unknown probability is now a random variable rather than a constant, we will denote by P rather than p). It can be shown that if one starts with a Beta prior distribution for P with parameters a and b and then in n experimental trials obtains s successes and f failures, then the posterior distribution of P is still a Beta, but with parameters a + s and b + f. Graphs of the PDFs and CDFs of several Beta distributions are shown in Section 1.3.

The "I-don't-have-the-vaguest-idea" prior distribution is a Beta with a = 1 and b = 1 which happens to also be a uniform distribution with l = 0 and h = 1. As Bayesians occasionally point out to their recalcitrant classical colleagues, in some ways using this prior is essentially what a classicist does all the time even though the nature of the experiment strongly suggests a more restricted range. For example, when tossing a coin to estimate the probability of a head, we would be surprised if the value of the estimate was very far from 1/2; consequently, the choice of the [0,1] uniform as a prior distribution seems a bit bizarre.

For some purposes, uniform distributions defined on subintervals of [0,1] make useful prior distributions. For example, the uniform on [0.4, 0.6] might be a good "It's-pretty-close-to-1/2" prior for coin tossing. Its simplicity makes it easier to select than a Beta and well suited for illustrative examples (as we will see in Section A.4).

A.4 ANALYSIS OF SYSTEMS.

To illustrate in some detail the analysis of good/bad systems, we will consider two simple examples: a two-component series system and a two-component parallel system. In each case the components are good/bad, and our problem is to investigate the probability of success of each system. We assume that each system is not testable. Consequently, any information about the system success must come from knowledge of the individual components.

Table A-11, Simple Series and Parallel Systems Event Table, defines the logical structure of the two systems. The two components are denoted by C and D (we will use the same ones for both systems), the systems by S (Series) and L (parallel), and the system or component states by g (good) and b (bad).

Table A-11. Simple series and parallel systems event table.

Component Events		System Events	
C	D	S	L
g	g	g	g
g	b	b	g
b	g	b	g
b	b	b	b

We will symbolize the event "C is good" by Cg and similarly for the other ones. Our ultimate interest is in what can be said about the probability that S is good -- Pr(Sg) -- and the probability that L is good -- Pr(Lg). These probabilities are either unknown constants or random variables with unknown distributions, depending whether we are talking Classic or Bayesian.

A.4.1 System Probability Formulas.

What we are searching for here is a formula or some other algorithm that gives the values of the success probability of each system in terms of the success probability of the components -- that is, Sg and Lg each in terms of Cg and Dg. Because of the simplicity of the systems, formulas are quite easy to obtain by starting with Event Table A-11.

For the series system the only outcome that produces Sg is the one in which both C and D are good. Therefore,

$$Sg = Cg \text{ and } Dg$$

and so

$$\begin{aligned} Pr(Sg) &= Pr(Cg \text{ and } Lg) \\ &= Pr(Cg) \times Pr(Dg|Cg). \end{aligned}$$

If we have no specific knowledge of the joint behavior of C and D and have no suspicion that their probabilistic behaviors are not independent, we will assume independence and get

$$Pr(Sg) = Pr(Cg) \times Pr(Dg)$$

We see from the event table that the parallel system is good if any of the top three events in the table occur. Consequently,

$$Pr(Lg) = Pr[(Cg \text{ and } Dg) \text{ or } (Cg \text{ and } Db) \text{ or } (Cb \text{ and } Dg)]$$

which, because the three "and" events are mutually exclusive, becomes

$$Pr(Lg) = Pr(Cg \text{ and } Dg) + Pr(Cg \text{ and } Db) + Pr(Cb \text{ and } Dg)$$

and, still assuming independence

$$\begin{aligned} Pr(Lg) &= Pr(Cg) \times Pr(Dg) + Pr(Cg) \times Pr(Db) \\ &\quad + Pr(Cb) \times Pr(Dg) \end{aligned}$$

This formula can be simplified by replacing Pr(Cb) by 1 - Pr(Cg) and Pr(Db) by 1 - Pr(Dg). Doing this, we can reduce the formula to

$$Pr(Lg) = Pr(Cg) + Pr(Dg) - Pr(Cg) \times Pr(Dg).$$

(There are several other ways of obtaining this formula, but we won't belabor the point.)

Explicit formulas for system probabilities are very nice to have but for most large and complex systems are extremely difficult to obtain. Fortunately, computer power and the availability of appropriate programs have reduced the need for formulas.

We observe that we could have added another column to the event table that would contain the probability associated with that row -- for example, the entry for the first row would be Pr(Cg and Dg) or, assuming independence, Pr(Cg) x Pr(Dg). Then it would be the simple matter of just adding the appropriate entries to get the

required system probabilities. A more pleasing presentation of this idea is made by using the event tree in Figure A-4 rather than the event table. In addition to a few obvious presentation changes, we have symbolized Pr(Cg) by "c" and Pr(Cb) by "1-c" (and similarly for other probabilities) and have written them on the "twigs" which connect the events. With this, the probability for each "branch" is simply the product of the probabilities written on the twigs belonging to the branch. The branches which produce the events Sg and Lg are marked with asterisks.

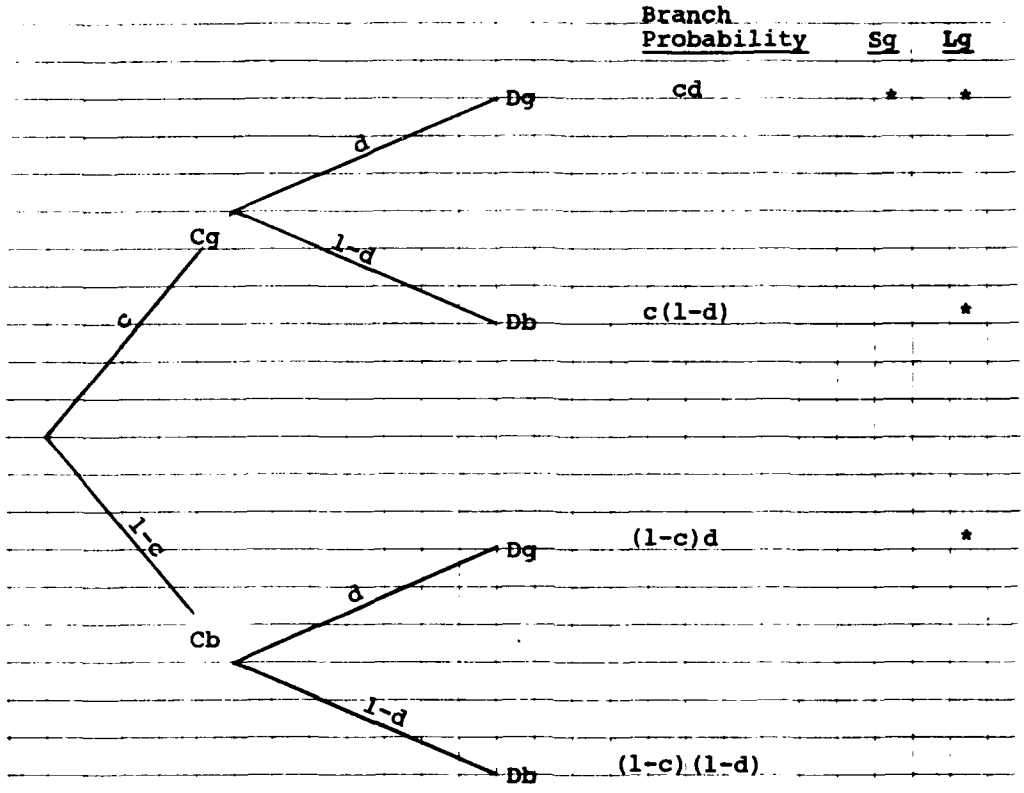


Figure A-4. Simple series and parallel systems event tree.

The most interesting thing about an event tree is that the idea is perfectly general and can be used with any system each of whose components have a finite number of outcomes. Of course, when many components are involved, the tree becomes quite unmanageable for most people, but there are existing computer programs that, by various methods, can handle systems with several hundreds or thousands of components. For example, GO does this by combining branches at judicious points and pruning branches with very small probabilities.

A.4.2 Component Probabilities With Classical Estimates.

If point estimates of the success probabilities of components are available, then a point estimate of the success probability of a system can be made by using a formula as we did in Section A.4.1. However, the problem of estimating the variability of the system point estimates using estimates of the component probability variabilities is almost always difficult, and the results are usually less than satisfying because of the need for simplifying assumptions. For certain types of system structures, reasonable

methods have been developed over the years, but for others -- particularly large and complex ones -- satisfactory procedures are generally not available. For this reason, we feel that turning to the Bayesian viewpoint is appropriate, particularly in view of the ready availability of the computing power needed for most Bayesian simulation studies.

A.4.3 Component Probabilities: Bayesian Estimates.

A Baye's distribution for the probability of an event contains all the current information about that probability. This information is fact-based as much as possible, but also belief-based which allows the experience of experts to be reflected in the distribution. Of course, because of different experiences, different experts may come up with different distributions for the same probability even if they are working with the same experimental test data, but if using the two distributions produces significantly different results in a system study, at least the source of the discrepancy is clear, and the matter can presumably be resolved by further tests and/or discussions.

Although the Baye's distributions for the components of a system can occasionally be mathematically manipulated to produce one for the system, this generally is not possible. Consequently, Monte Carlo simulation is the method that is commonly used. Conceptually, this amounts to obtaining a specific success probability for each component by sampling from each of the component distributions and then using a formula or other algorithm to compute the system success probability. This process is repeated many times (usually several hundred at least), and the resulting collection of system success probabilities serves as an estimate of the actual system success probability distribution.

The sampling of a component distribution is done by the method described in Section A.2.8 in which a random number from the [0,1] uniform distribution is transformed into a random number from the desired distribution by using the inverse of that distribution's CDF.

To illustrate the process for our two systems, we assume that pr(Cg) and Pr(Dg) are random variables which we will denote by C and D. To keep the arithmetic at an elementary level we will use uniform Baye's distributions. In particular, we assume the PDFs of C and D are

$$PDF_C(x) = 1/(0.6 - 0.4) = 5, \quad 0.4 \le x \le 0.6,$$

$$PDF_D(x) = 1/(0.8 - 0.2) = 5/3, \quad 0.2 \le x \le 0.8,$$

which suggest that we are fairly sure that the values of C are close to 1/2 (the mean of C) but less sure for the values of D. The CDF of each distribution is obtained by integrating its PDF to give

$$CDF_C(x) = 5x - 2, \qquad 0.4 \le x \le 0.6,$$

$$CDF_D(x) = (5x-1)/3, \qquad 0.2 \le x \le 0.8,$$

and, finally, by some algebraic manipulation, the inverses of the CDFs are found to be

$$CDF_C^{-1}(y) = (y + 2)/5, \qquad 0 \le y \le 1,$$

$$CDF_{D'}(y) = (3y + 1)/5, \qquad 0 \le y \le 1.$$

All that we now need are the formulas for Pr(Sg) and Pr(Lg) which we obtained in Section A.4.1 and a source of uniformly distributed random numbers between 0 and 1. Letting c and d represent sampled values of C and D, the formulas become

$$Pr(Sg) = cd \quad \text{and} \quad Pr(Lg) = c + d - cd.$$

There are many sources for the random numbers, but a computer is the common provider today (we used a Tandy PC-6 pocket computer for the ones here).

Table A-12 shows the results of five samplings of each of the component distributions and the computation of the systems' success probabilities. In the table r is a random number value from the Tandy and s and l are the resulting sample values of S = Pr(Sg) and L = Pr(Lg).

Table A-12. System estimates for simple series and parallel systems.

Component C		Component D		Systems	
r	c	r	d	s	l
.14	.43	.80	.68	.29	.82
.64	.53	.47	.49	.26	.76
.26	.45	.23	.34	.15	.64
.16	.43	.01	.21	.09	.55
.72	.54	.50	.50	.27	.77

We can now obtain an estimate of the Baye's CDF of S by ordering the s values and estimating the value of $CDF_S(s)$ as the proportion of the sample size (5) of the number of sample values less than or equal to s -- that is, the number of such values divided by the sample size. The estimated CDF and its graph are shown in Figure A-5.

The two columns containing values of r in Table A-12 represent estimates of the uniform Baye's distributions of C and D. Table A-13 tabulates the data for the sample estimates of the CDFs of C,D,S, and L (note that the ordinates are the same for each because the sample sizes are all the same), and Figure A-6 shows the graphs of these functions. We have connected the points on each graph with straight line segments instead of using the customary steps. The latter would be a correct representation of a sample CDF itself, whereas the former is a better estimate of the parent CDF.

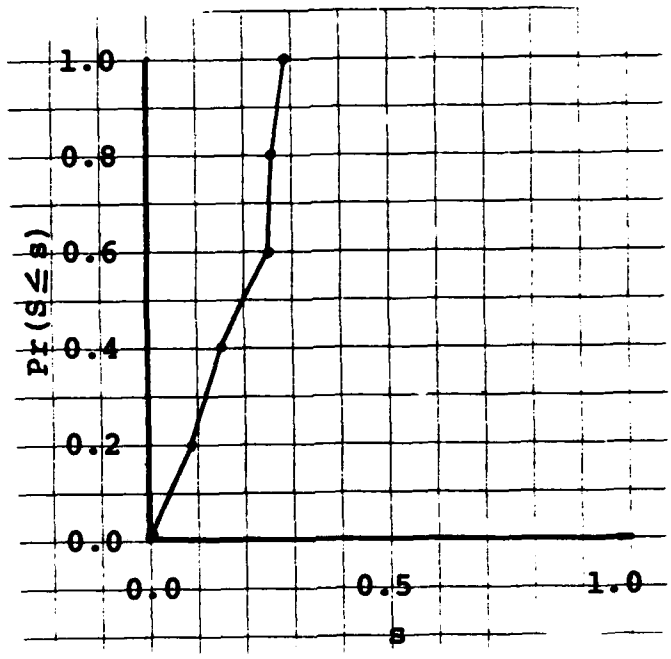


Figure A-5. Sample CDF for series system (n = 5).

Table A-13. Sample data for CDF's.

c	d	s	l	Ordinates
0.43	0.21	0.09	0.55	0.2
0.43	0.34	0.15	0.64	0.4
0.45	0.49	0.26	0.76	0.6
0.53	0.50	0.27	0.77	0.8
0.54	0.68	0.29	0.82	1.0

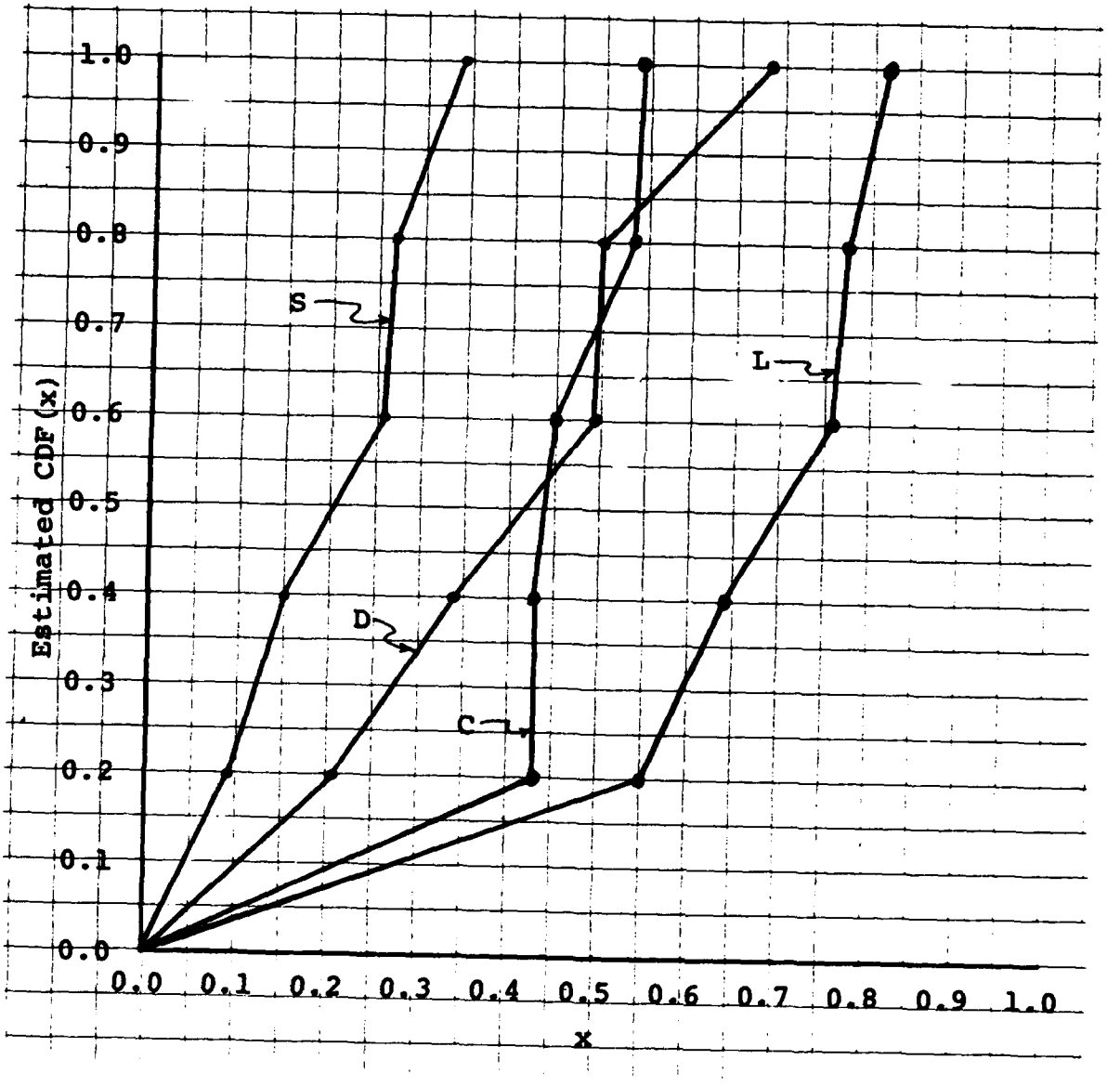


Figure A-6. Sample CDFs for C, D, S, and L (n = 5).

Figure A-7 shows the graphs of the sample CDFs of C,D,S, and L for a sample size of 500 (the computing was done with a 45-line BASIC program on a 286 microcomputer). As mentioned earlier, graphs of the parent CDFs of C and D are straight line segments, while those of S and L are unknown.

From these graphs we can obtain a variety of estimated probability intervals. For example, the observation that

$$\Pr(S \leq 0.15) = 0.10$$

gives us (0,0.15) as a 10% probability interval for S, and

$$\Pr(0.15 \leq S \leq 0.40) = 0.80$$

gives (0.15, 0.40) as an 80% interval.

Noting that

$$\Pr(S \leq 0.15) + \Pr(S > 0.15) = 1.0$$

we have

$$\begin{aligned} \Pr(S > 0.15) &= 1.0 - 0.10 \\ &= 0.90, \end{aligned}$$

which says that (0.15, 1.0) is a 90% interval for S. The adjective "survivability" might be applied to an interval of this kind, and flipping the CDF graph upside-down and relabeling the vertical axis to read "$\Pr(S \leq s)$" gives us the graph of a survivability function.

The observation that the CDF of S falls to the left of those of C and D and that the CDF of L falls to the right is of considerable interest because it is an example of the general statement that the probability performance of a series system is always worse than that of its worst components, and that the performance of a parallel system is always better than that of its best component.

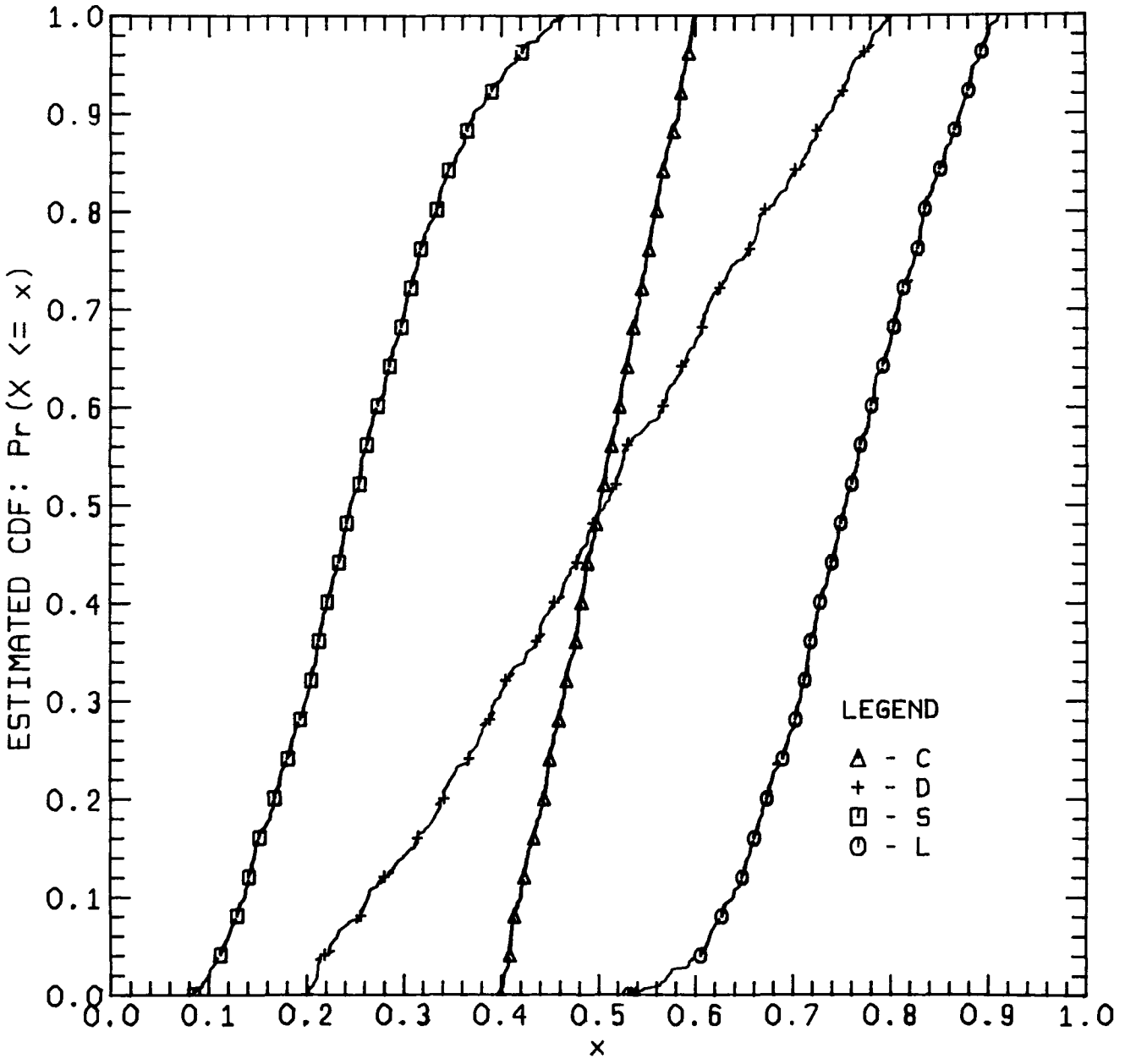


Figure A-7. Sample CDFs for C, D, S, and L (n = 500).

It is important to remember that the ordinates of the plotted points on graphs such as these are estimates of the unknown CDF values. The word "estimate" will frequently be missing when these matters are being discussed, and its omission can easily mislead a person into assuming that he is seeing the truth rather than an estimate of the truth.

This cautionary statement brings up the question of how close the estimate is to the truth. We are certainly inclined to have more faith in Figure A-7 with $n = 500$ than in Figure A-6 with $n = 5$. Because the CDFs of C and D are known (we had to use their inverses to do the sampling), we can see the actual differences between the truth and the estimates for these cases. Figure A-8 shows the real CDF of D (labeled $n = \infty$) and the sample estimates for both $n = 5$ and $n = 500$. A similar figure for C would show better fits because C has a smaller variance than D.

We are, of course, unable to plot the real CDFs of S and L because we don't know what they are. In a very few special cases such information could be obtained analytically, but there is no general procedure for doing this.

There is, however, something which can be said. The use of the Kolmogorov (or Kolmogorov-Smirnov) random variable allows us to place two curves around a sample CDF such that there is a specified confidence that the band between these curves contains the real CDF. The Kolmogorov random variable can be used for any CDF and, consequently, is referred to as being "distribution free" or "non-parametric". The values of the random variable (k) for a given sample size (n) and a given confidence (c) can be found in tables in many statistics books, but it can be computed accurately by the formula

$$k = [-\ln((1-c)/2)/(2n)]^{(1/2)}$$

The formula is approximate, but, except for very small sample values, gives excellent results.

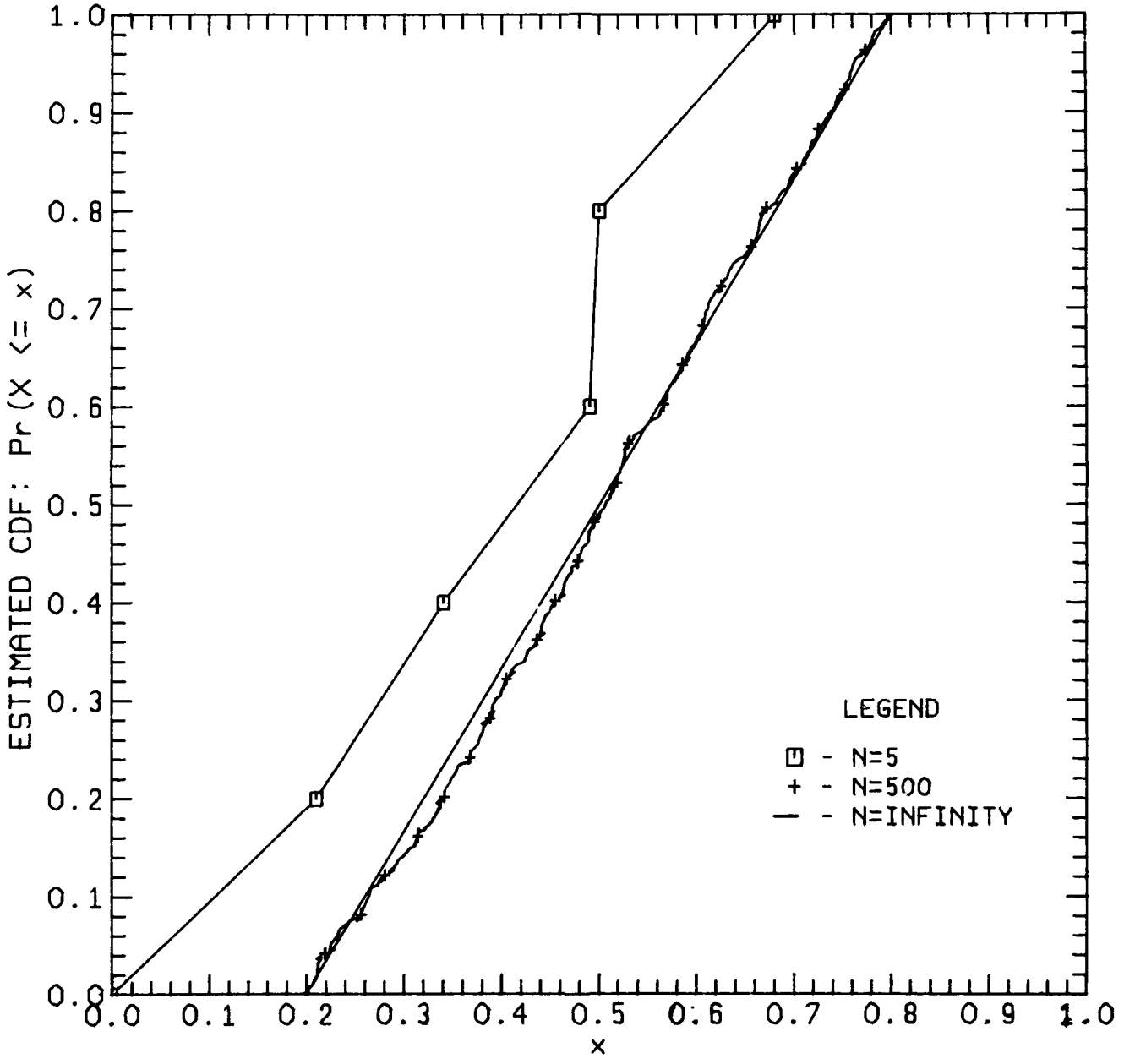


Figure A-8. Real CDF for D.

The upper bound of the confidence band is constructed by adding the value of k to the ordinate of each sample point, but replacing the new ordinate by 1 if it exceeds 1, and then connecting the new points in the same way as the sample points are connected. The lower bound is found in a similar manner by subtracting k from the ordinate of the sample points. Some values of k that could be applied to our examples are given in Table A-14. The table confirms the basic nature of all confidence statements: the higher the confidence, the less there is to be confident about.

Table A-14. The Kolmogorov-Smirnov random variable.

Sample Size				
Confidence	5*	50	500	5000
0.90	0.518	0.173	0.055	0.017
0.95	0.563	0.192	0.061	0.019
0.99	0.669	0.230	0.073	0.023

* The entries for n = 5 are from tables; the formula gives slightly higher values for small n.

A.4.4 Using CDF Estimates.

Having obtained an estimate of the CDF of the success probability of a system, what can we do with it?

- By itself the sample CDF can give us a good idea of what can be expected from the system. If the variability is large, the CDF will be tilted more than if the variability is small (Figure A-6 indicates that C has a much smaller variability than D). A large variability indicates that there are some substantial uncertainties that should be reduced if at all possible.

- The CDF can be used as an accept/reject tool if the requirements for the system have been stated in an appropriate form -- for example, the success probability of the system must be at least 0.96 at a 90% confidence level.

- If the system becomes a part of a larger one, the sample CDF can be used to generate random values for use in the larger system.

A.4.5 Fragility Functions.

A component of a system is not always of the good/bad kind, but instead, may produce an output that is a continuous random variable -- such as a force -- which may cause a failure in a following component. The relationship between the value of the random variable and the failure probability of the impacted component is called a fragility function. For example, consider the experiment of dropping an egg from a certain height onto a carpeted floor and seeing whether or not the egg breaks. There is certainly some height below which an egg will always survive and some height above which it never will. At some fixed height between these extremes sometimes an egg will break and sometimes it won't, depending

presumably on just how it lands and how well-built the egg was to start with. By numerous tests, extensive analysis, and probably some graybeard intuition, a fragility function can be determined and might look something like Figure A-9 which shows the graph of Pr(Egg Survives) versus the dropping height. (We could of course turn the curve upside-down and relabel the vertical axis "Pr(Egg Breaks)."

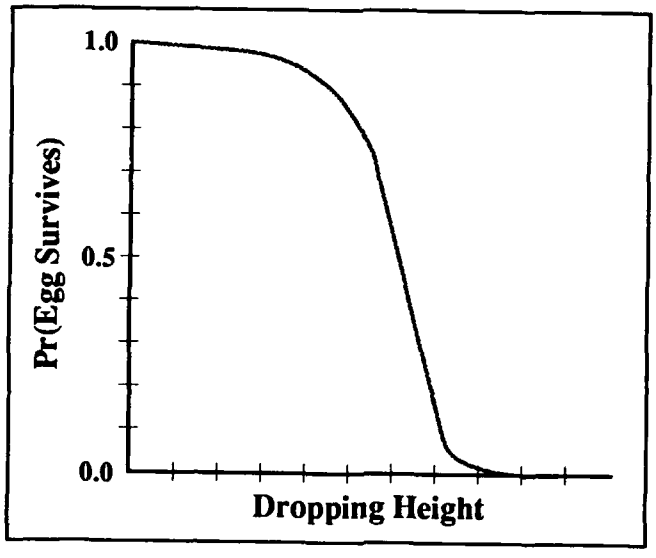


Figure A-9. Example fragility function.

If we now have a system which involves lifting an egg and then dropping it, and the height to which it is lifted is a random variable, we can simulate the system performance by sampling the height distribution and using the fragility function to determine the resulting survival probability. Repeating this process many times produces an estimate of the CDF of this probability.

A.4.6 Uncertainties.

In this section we discuss some of the uncertainties that befuddle our best efforts to understand and evaluate the performance of a system, large or small. There is no way in this life that we can eliminate all uncertainties, but it is important to be aware of as many of them as possible so that any efforts to reduce them can be properly allocated.

A.4.6.1 Modeling Errors. The results of a performance analysis apply to the system model used and to the system itself only if the model is a faithful representation of the system insofar as the particular analysis is concerned. It is probably safe to say that the creation of a model of a large system is the most difficult part of a performance study and the one most likely to produce wrong answers. The most meticulous and exhaustive study using an incorrect model will never produce correct results for the modeled system.

A bad model is usually hard to detect because in most cases a lot of time and effort has been expended in its creation. Every effort must be made to compare model results of any kind against those produced by the real system. A discrepancy

suggests either changing the model to produce the proper results, or, if the model results are preferable to those of the system, changing the system to match the model (the latter is why many models are built in the first place).

A.4.6.2 Random Error. Random errors are simply those variations in the performance of a system which are irreducible. By testing we are usually able to describe the behavior of such errors in terms of a probability distribution, but it is never possible to predict the particular value that will occur the next time the system operates. The effects of random errors can sometimes be overcome by using redundancy in a system; we can create a more reliable system if we can connect its unreliable components in parallel. On the other hand, using many reliable components in series tends to create unreliable systems.

A.4.6.3 Systematic Uncertainties. Systematic uncertainties are those which are associated with an increased variability, but which we are aware of and can presumably reduce in one way or another. For example, measurements of almost any kind involve the object being measured, the measuring instrument, and the person doing the measuring. All three of these may produce some random errors, but the instrument and the measurer may also contribute systematic uncertainties (biases) -- for example, an imprecise or inaccurate instrument or an unskilled measurer, both of which can be improved if need be.

A fragility function will almost certainly involve both random errors and systematic uncertainties -- there are always more experiments which ought to be done in order to improve a formula or an algorithm.

It is, of course, highly desirable to identify the existence of systematic uncertainties and to analyze their effects upon the system performance. Traditional statistical analysis of variance techniques may be useful for this in some situations.

In Monte Carlo simulations and component tests some uncertainty is introduced because of sample size limitations. The question of how many simulations or tests are enough is always pertinent, but is frequently ignored. We have discussed these problems to some extent earlier and will reiterate that they are always with us, but are amenable to study, and, given the necessary resources, reducible.

A.4.7 Large System Analyses.

The analysis (usually a simulation) of a small system is generally a simple and straightforward procedure. A system that contains many components that are connected together in a complex manner can present a much more serious problem. Fortunately, our analysis tool box now contains large and fast computers and many special computer programs for doing much of the work. However, the creation of an adequate system model, and the creation of additional computer programs to handle non-standard situations still requires a great deal of human expertise. There is no single tool that will serve all kinds of system models, but there is no obvious reason to doubt that any system can be modeled and analyzed. This is not to say that the resulting answers will be what is desired or expected because the cost and/or difficulty of extensive component testing will, in most cases, require prior distributions with considerable variability, and these will probably not produce comforting results.

APPENDIX B

GLOSSARY OF TERMS

AGT - aboveground test

Bayesian - one who believes that any quantity whose value he does not know is a random variable and that it is possible to express his current knowledge about such a random variable in the form of a PDF or PMF

BMEWS - Ballistic Missile Early Warning System

Central Limit Theorem - the sum of a large number of independent random variables has approximately a normal distribution regardless of what distribution the summands have

CDF - cumulative distribution function: $Pr(X \geq x)$

CINCSPACE - Commander In Chief, Space

Component - an element of a system comprising one or more piece parts

Conditional Distribution - the resultant probability distribution obtained from a joint probability distribution given the specified value of one or more other random variables

Conditioning Event - the event whose occurrence is postulated to occur before finding the conditional distribution

Confidence Interval - an interval within which the value of a random variable is likely to lie with specified probability

Continuous Random Variable - a random variable that takes values within a continuous set of real numbers

Discrete Random Variable - a random variable that takes only a discrete set of values

Distribution-free - no distribution form has been assumed for the random variable of interest

DSP - Defense Support Program

Expected Value - see mean

Experiment - exercising the system one or more times to observe the result

Fragility Function - a function relating the probability of catastrophic damage to a component as a function of the stress imposed

Fundamental Theorem of Calculus - $F'(x) = f(x)$ and $\int_a^b f(x)dx = F(b)-F(a)$

GO - a probabilistic analysis procedure and software for modeling systems to develop information about system performance from similar data about the

constituent components. Models are success-oriented and easy to develop and validate. The basic GO procedure is supplemented with additional software to identify fault sets and to place confidence bounds on system event estimates as a function of data uncertainties. (See References)

GO type-kind numbers - type: one of 17 logical operators in GO; kind: a number associating probabilities in a data file to the component operation states

GBI - Ground-Based Interceptor

GBR - Ground-Based Radar

GPALS - Global Protection Against Limited Strikes

GSTS - Ground-Based Surveillance and Tracking System

Interval Estimate - an interval in which the values taken by a random variable are likely to be, usually associated with a probability

Kolmogorov-Smirnov Random Variable - defined as a function of confidence and sample size permitting the construction of a distribution-free band about a sample CDF containing the true CDF with specified confidence

Law of Large Numbers - as the number of samples becomes large the variation between the sample mean and the true mean becomes arbitrarily small

LCB - Lower Confidence Bound

Marginal Distribution - the distribution for a single variable after summing (or integrating) out all other random variables in a joint probability distribution

Mean - the first moment about the origin of any PDF or PMF: symbol m

Model - a representation of some aspects of a system

Monte Carlo Simulation - random sampling from known distributions

Mutually Exclusive Events - two events are mutually exclusive if the occurrence of either precludes the occurrence of the other

NCA - National Command Authority

NMD - National Missile Defense

Non-parametric - no distribution form has been assumed for the random variable of interest

Outcome - a fundamental result of an experiment

Parameter - constants that can take various values to define specific distributions within a class

PDF - probability density function (defined for continuous random variables)

Pk - probability of kill

PMF - probability mass function (defined for discrete random variables)

Point Estimate - a single value said to be representative of values taken by a random variable: often specific estimators are used (e.g., the mean)

Posterior Distribution - the distribution resultant after additional information is provided

Probability Interval - Bayesian terminology: same as a confidence interval

Prior Distribution - the distribution assumed by a Bayesian before additional information is provided

Random Drawing - each member of the population has an equal chance of being drawn on each trial

Random Error - irreducible experimental scatter

Random Sample - a set of observations obtained from a random drawing from a population

Random Variable - a variable whose values range over a set of real numbers

Sample Space - the set of possible outcomes from an experiment

S.D. or SD - standard deviation; square root of variance: symbol σ

Standard Normal Probability Distribution - a normal distribution with mean 0 and S.D. 1 into which all other normal distributions can be transformed

Statistical Independence - two events are statistically independent when the occurrence of one in no way influences the occurrence of the other

Subsystem - an element of a system comprising several components

Survivability - the probability that a component or system survives a specified threat

Survivability Function - the inverse of the system survivability CDF

System - a collection of elements - piece parts, components, subsystems - designed to perform certain functions

Systematic Uncertainty - uncertainties or biases caused by lack of knowledge which can be reduced by additional tests and analysis

UGT - underground test

Validation - establishing by reason, simulation, test data, and procedures that a statement is valid

Variance - the second moment about the mean of any PDF or PMF: symbol σ^2

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